Minimax Theory and its Applications
Volume 11 (2026), No. 4s, 622 to 630

Inverse variational identification methods for hidden parameter estimation in nonlinear engineering dynamic models

Dr. Lakhwinder Kaur[1], S. Harthy Ruby Priya[2], R. Ramya[3], Dr.R. Kalpana[4], Baluvuri Sudhakara Rao[5], Dr. Akshit Lamba[6]

[1]Associate Professor, Department of Computer Science and Engineering, Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. lakhwinder.kaur@rungta.org, https://orcid.org/0000-0002-4939-3648

[2]Assistant Professor, Department of Computer Science and Engineering, J.J. College of Engineering and Technology, Tiruchirappalli, Tamil Nadu, India. harthyrubypriya@jjcet.ac.in, https://orcid.org/0009-0005-6218-3599

[3]Assistant Professor, Department of Electronics and Communication Engineering, K.S. Rangasamy College of Technology, Namakkal, Tamil Nadu, India. rramya@ksrct.ac.in, https://orcid.org/0000-0002-7841-4129

[4]Assistant Professor, Department of Management Studies, SRM Easwari Engineering College, Chennai, Tamil Nadu, India.

[5]Department of Mech, Ramachandra College of Engineering, Eluru, Andhra Pradesh, India. baluvurisudhakar@rcee.ac.in, https://orcid.org/0000-0002-9585-2294

[6]Assistant Professor, Kalinga University, Naya Raipur, Chhattisgarh, India. akshitlamba@kalingauniversity.ac.in, https://orcid.org/0009-0001-4148-6081

Abstract:

Hidden parameter estimation in nonlinear engineering dynamic systems is seen as an ill-name inverse variational problem instead of just being a direct least-squares fit. In other words, a cost functional that penalizes trajectory mismatch and then minimizes this cost function under a system of nonlinear dynamic constraints and inequality bounds defined by physical admissibility conditions, such as stresses, displacements and damping limits ensuing from physical admissibility constraints. Then, using a Lagrangian duality argument, the study reduces constrained program to a standard minimax problem formulation and introduces an appropriate characterization of stationarity by means of generalized Karush-Kuhn-Tucker conditions based on Clarke subdifferential calculus allowing one to deal with non-smooth boundary conditions. The study shown that through the use of generalized Mangasarian-Fromovitz conditions along with metric subregularity of the constraint map, one can guarantee both boundedness and local uniqueness of the set of Lagrange multipliers which prevents the pathological Lagrange multiplier blow-up as a consequence of failure of classical Linear Independence Constraint Qualification. The paper proposes a projected gradient minimax algorithm and proves a descent lemma under bounded measurement noise establishing linear convergence of the algorithm to the hidden parameter vector. Then apply our approach to a nonlinear engineering example of hidden stiffness coefficients and compare the proposed framework with classical formulation in terms of numerical stability of multiplication norm as well as the convergence of the parameters and error of the trajectory.

Keywords: Inverse variational problems, hidden parameter estimation, minimax duality, metric subregularity, generalized mangasarian–fromovitz qualification, non-smooth Lagrangian analysis, nonlinear system identification.

1. Introduction

1.1. Background and motivation

Nonlinear dynamic systems, including construction systems, electro-mechanical drives, robotic arms, and connected thermo-mechanical systems, are often described by state equations in which a number of coefficients cannot be measured directly. Such coefficients include degradation coefficients, hidden stiffness coefficients, friction coefficients, and damping coefficients, and usually included in the equation in a multiplicative way or with the help of non-smooth switch-like terms, so only indirect information about them can be obtained from the sensors' measurements. Variational identification of the linearly parameterized nonlinear system has been defined as a maximum likelihood consistent problem with proof of statistical properties (Liu & Yang, 2023). The corresponding path-planning and navigation problems based on reinforcement learning highlight that hidden decision-relevant parameters can be found indirectly using the optimization process aimed at maximizing an appropriate reward function (Kappagantula & Mannayee, 2024). Invertibleinference technology built on invertible neural networks was used to find the unknown hemodynamic parameters as well as to make identifiability analysis of the underlying lumped parameter model (Tong et al., 2025), while the cost selection technologies based on neural networks confirm the viability of the usage of learned surrogates in engineering and economic problems (Agrab, 2022).

1.2 Inverse problems and variational formulations

Defining the problem of casting parameter recovery as least-squares fitting is effective but it masks the difficulties caused by the deficiencies present within the inverse mapping inherently: small perturbations in sensor measurements can lead to disproportionate deviations in the parameter vector recovered. This paper shows that enforcing the stability of the estimation task through constraint qualification allows a better formulation of the problem of estimation through the regularized inverse problem; the burden of regularization is no longer loaded only on the regularization parameters. The idea of combining variational approximations of states and noise covariances has been proven to significantly improve the conditioning of the problem compared with the sequential approach (Lan et al., 2024). The neural forecasting models developed for studying financial time series serve as an example of the idea that the unobservable causes of the process can be tackled through optimization methods even outside mechanical engineering applications (Mousavi & Karshenasan, 2017). The review of the literature on the problem of passive identification in robotics proves the fragile nature of classical formulations in situations of failures of constraint qualifications (Leboutet et al., 2021). The application of variational states-space identification to skewed and heavy-tailed noise demonstrates that the assumption about distribution of the noise has a strong impact on the conditioning (Liu & Yang, 2022). Formal verification frameworks were explored for cloud-native microservices to improve system reliability, security, and correctness by validating service interactions and operational behavior in complex distributed environments (Patil et al., 2026).

1.3 Literature gap and contributions

Variational Bayes canonical correspondence analysis for continuous process control has shown that a stochastic variational layer enhances tracking of parameter related to failure during process evolution (Yu et al., 2021) and that process-variable identification across systems with different dynamics using variational approach indicates that parameters can be estimated across similar systems instead of being computed from zero (Ping et al., 2024). Transfer methods based on variational approach show that joint latent structure across different systems diminishes dimensionality of the problem (Xie et al., 2021). Nevertheless, the gap between the traditional definition of system estimation, which uses continuous least-squares residues and Linear Independence Constraint Qualification (LICQ), and its modern analog based on non-smooth analysis still exists. Three ways of successfully closing this gap are suggested. First, classical system estimation methods are connected with minimax duality and non-smooth optimization via unified saddle-point approach. Second, generalized Mangasarian–Fromovitz condition combined with metric sub-regularity of the constraint mapping is provided to ensure stability of the multiplier set under changing uncertainty, which is not the case under LICQ condition in the neighborhood of degeneracy. Finally, the classification of the estimation errors for the estimation of the hidden parameter is made under finite-noise assumption, which is proven numerically using a known example.

2. Problem formulation and geometric preliminaries

2.1. Nonlinear dynamic system representation

The engineering plant is represented in state-space form as Eq. (1):

$$\dot{x}(t) = f(x(t), u(t), \theta), \quad x(0) = x_0, \quad y(t) = h(x(t), \theta) \tag{1}$$

In Eq. (1), $x(t) \in \mathbb{R}^n$ indicates the state path that may not be fully calibrated. The control signal is presented in the form of $u(t) \in \mathbb{R}^m$ while $y(t) \in \mathbb{R}^p$ shows the final signal. The hidden parameters form vector $\theta \in \Theta \subset \mathbb{R}^k$ that belong to a permissible closed convex domain Θ reflecting the physical constraints like the existence of positive values of stiffness or damping coefficients. It is accepted that f has a local Lipschitz condition but it may have nonsmoothness with respect to θ in the moment of abrupt changes of the structure like switching due to the stick-slip friction or shift in the stiffness of a shipped body in the performance. The method of Bayesian estimation of the parameters based on the graph probabilistic model allows us to treat θ as random with the help of variational posterior approach rather than as the unknown provided in (Du & Katib, 2022), moreover, using the statistical mechanics perspective of identification of dynamic systems allows us to interpret the possibility of variables in the set Θ under the uncertainty of measurement (Klishin et al., 2025).

2.2. Inverse variational cost functional

The tracking objective is posed as a constrained variational optimization problem, given in Eq. (2):

$$\min_{\theta \in \Theta} J(\theta) = \int_0^T \| y(t) - \hat{y}(t; \theta) \|^2 \, dt + \lambda R(\theta) \quad \text{subject to} \quad g(x(t), \theta) \leq 0, \quad \dot{x}(t) = f(x(t), u(t), \theta) \tag{2}$$

Here, in Eq. (2), $\hat{y}(t; \theta)$ is the predicted output of the model, whereas R represents the regularization term for stabilizing inverse mapping which is otherwise ill-defined; $\lambda > 0$ is the weight of regularization; and g refers to the inequality constraints constituting physical admissibility (limit of stress, bound of displacement, limit of actuator). Solving forward and inverse problems using machine learning for estimation of wildfire propagation parameters proves the existence of an analogous cost functional structure in the area of distributed parameters constituting forward modeling and inverse calibration (Cheng et al., 2022). Furthermore, a wide literature summary on data assimilation coupled with machine learning in the context of dynamics systems allows us to place the cost functional of Eq. (2) in a wider uncertainty quantification context (Cheng et al., 2023).

2.3. Geometric cones and constraint qualifications

Stability of the multiplier set associated with the constraints $g(x, \theta) \leq 0$ is analyzed through the Bouligand tangent cone, given in Eq. (3):

$$T_\Theta(\theta^*) = \{d \in \mathbb{R}^k : \exists t_n \downarrow 0, d_n \to d \text{ with } \theta^* + t_n d_n \in \Theta\} \tag{3}$$

and the linearized feasible cone, given in equation (4):

$$L(\theta^*) = \{d \in \mathbb{R}^k : \nabla g_i(\theta^*)^T d \leq 0 \text{ for all } i \in I(\theta^*)\} \tag{4}$$

where $I(\theta^*)$ refers to the active constraints at θ^*. A generalized Mangasarian-Fromovitz constraint qualification (GMFCQ) is said to hold at θ^* if $T_\Theta(\theta^*) = L(\theta^*)$, i.e., if equation (3) coincides with equation (4), and establishes uniform metric subregularity, a condition where a metric subregularity bound exists, given in equation (5):

$$\text{dist}(\theta, \{\theta' : g(\theta') \leq 0\}) \leq \kappa \| \max\{g(\theta), 0\} \| \quad \text{for all } \theta \text{ near } \theta^* \tag{5}$$

for some $\kappa > 0$, avoiding issues with unbounded Lagrange multipliers that arise from LICQ failure at the points of active constraint linear dependence. As guaranteed by Eq. (5), invertible liquid neural networks for learning inverse kinematics and dynamics of robotic manipulators illustrate the practical consequences of qualification failure, where naive inversion becomes numerically unstable at kinematic singularities, the occurrences of which correspond to constraint degeneracy in the given geometric sense (Zhang et al., 2025). A survey on system identification in the network era provides information on the availability of data, making qualification failure worse in cases of uncertain or sparse sensor network connectivity (Wang & Zhang, 2025). Bayesian inverse mapping models for real-time parameter refitting of digital twins demonstrate that metric subregularity-type stability conditions of the form in Eq. (5) are vital for guaranteed reliability of online recalibration under

continuously delivered noisy data for the processing algorithm (Kessels et al., 2025). The neural extended Kalman filter gives an example of how the models of dynamics learn the same stability, depending on qualification, while parameter updates are integrated into filter recursive process (Liu et al., 2024). Variational neural networks using planar normalizing flows for inertial-GPS trajectory merging showcase one more case where the flexible non-smooth posterior needs precisely the sort of metric subregularity-type control introduced in Eq. (5) to remain well-posed (Fu & Jin, 2025). Amortized Bayesian estimation of parameters of the composite load models in power engineering indicates that multiplier instability may happen with classical qualification methods due to the total degeneracy of the active constraints set at the operating points (Tan et al., 2023), while physics-informed inverse problem solving methods applied to maglev systems with noisy measurements reveal that GMFCQ conditions improve the results of hidden parameter reconstructions as compared with unconstrained neural inversion methods (Ni et al., 2025).

3. Variational identification via minimax and duality frameworks

3.1. Saddle-point reduction

The constrained optimization issue that was defined in Section 2.2 (Eq. 2) turns into the locally limitated minimax programming situation stated in Eq. (6) when the multiplier function $\mu(t) \geq 0$ is applied to inequality constraint $g(x(t), \theta) \leq 0$ and the costate $\psi(t)$ corresponds to the dynamic's constraints.

$$\min_{\theta \in \Theta} \max_{\mu \geq 0, \psi} \mathcal{L}(\theta, \mu, \psi) = J(\theta) + \int_0^T \mu\,(t)^T g(x(t), \theta)\, dt + \int_0^T \psi\,(t)^T [f(x(t), u(t), \theta) - \dot{x}(t)]\, dt \tag{6}$$

With the conditions of GMFCQ and metrics subregularity described in Eqs. (3) - (5) being satisfied, the strong duality is ensured locally and the saddle point's outcome of Eq. (6) matches primal problem outcome of constraint inverse task which allows minimizing θ unconstrainedly (in terms of μ and ψ).

3.2. Generalized KKT conditions for hidden parameters

Since g and f might not be smooth with respect to θ, stationarity is represented using Clarke's generalized gradient ∂g instead of the classical gradient. A point $(\theta^*, \mu^*, \psi^*)$ is called a generalized KKT point if the equation (7) holds:

$$0 \in \partial_\theta J(\theta^*) + \sum_{i \in I(\theta^*)} \mu_i^*\, \partial_\theta g_i(\theta^*, x^*) + \partial_\theta \langle \psi^*, f(x^*, u, \theta^*) \rangle, \quad \mu_i^* \geq 0, \quad g_i(\theta^*, x^*) \leq 0, \quad \mu_i^* g_i(\theta^*, x^*) = 0 \quad \text{for all } i \tag{7}$$

As stated in equation (7), the subdifferential formulation allows for sudden structural changes, for example, stiffness reduction activated by a threshold, without any requirement of smoothing approximations which would induce bias in the reconstructed hidden parameter.

3.3. Error bounds and metric subregularity

Taking into consideration the metric subregularity criterion concerning the mapping $G(\theta) = \max\{g(\theta), 0\}$ expressed in (5), get a bound for the inverse residual in the following form (8):

$$\|\theta - \theta^*\| \leq \kappa \left(\|G(\theta)\| + \operatorname{dist}\big(0, \partial_\theta \mathcal{L}(\theta, \mu^*, \psi^*)\big) \right) \tag{8}$$

for θ close enough to θ^*, where κ characterizes the subregularity index. The bound (8) allows for understanding convergence stability, given that when the generalized stationarity residual and its violation are kept small at the same time, one can control the distance to the real hidden parameter regardless of the non-unique nature of the multiplier pair (μ^*, ψ^*).

4. Computational algorithm and convergence analysis

4.1 Projected-gradient minimax interior-point scheme

The saddle point of Eq. (6) is approximated through the coupled iteration given in equation (9):

$$x_{k+1} = x_k + h \cdot f(x_k, u_k, \theta_k) \quad \text{(state propagation)}$$

$$\theta_{k+1} = \text{Proj}_{\Theta}\left[\theta_k - \alpha_k\left(\nabla_\theta J(\theta_k) + \sum_i \mu_{k,i}\,\nabla_\theta g_i(\theta_k)\right)\right] \quad \text{(parameter update)}$$

$$\mu_{k+1} = \max\{0, \mu_k + \beta_k g(x_k, \theta_k)\} \quad \text{(multiplier ascent)} \tag{9}$$

Where α_k and β_k represent the step sizes that comply with the diminishing-step conditions and Projector onto admissible set. To make the projection smoother, the interior-point barrier function is used in g that is close to boundary of the active constraint, as explained in Eq. (9).

4.2. Second-order sufficiency conditions

The critical cone at a generalized KKT point θ^* is defined in Eq. (10) as

$$C(\theta^*) = \{d \in T_\Theta(\theta^*): \nabla_\theta g_i(\theta^*)^T d = 0 \text{ for } i \text{ with } \mu_i^* > 0\} \tag{10}$$

and strict positive definiteness of the Hessian of the Lagrangian restricted to this cone is required, as stated in Eq. (11):

$$d^T \nabla^2_{\theta\theta} \mathcal{L}(\theta^*, \mu^*, \psi^*) d > 0 \quad \text{for all } d \in C(\theta^*) \setminus \{0\} \tag{11}$$

According to equation (11) θ^* is identified as being a locally unique and stable estimator for the hidden parameter vector and avoids existence of flat or saddle points of the cost function in which θ cannot be uniquely determined.

4.3. Convergence proof sketch

A descent lemma is applied to the augmented Lagrangian, defined in equation (12) as:

$$\mathcal{L}_\rho(\theta, \mu) = \mathcal{L}(\theta, \mu) + \frac{\rho}{2}\left\|\max\left\{g, -\frac{\mu}{\rho}\right\}\right\|^2 \tag{12}$$

$$\|\theta_{k+1} - \theta^*\| \le (1 - c\alpha_k)\|\theta_k - \theta^*\| + \alpha_k \varepsilon_k \tag{13}$$

Hence, it can be deduced that if proper steps are taken such that $\alpha_k < \frac{2}{\rho \cdot \text{Lip}(\nabla_\theta g)}$, the sequence $\{\mathcal{L}_\rho(\theta_k, \mu_k)\}$ defined in Eq. (12) will start a non-increasing sequence when combined with the bounding error given in Eq. (8) leading to the convergence property given in Eq. (13), where ε_k stands for the bounded sensor noise which appears at the iteration k and $c > 0$ is defined only through subregularity modulus value κ given in Eq. (5) and the minimum eigenvalue of the Hessian which is defined in Eq. (11).

5. Application to a nonlinear engineering benchmark

5.1. Case study description

The method has been tested on a structural dynamics test problem represented by a multi-rod truss structure characterized by coordinated stress–displacement limits, where the magnitudes of the two rigidity factors (θ_1 and θ_2) of a group of rods are unknown and should be determined based on measurement data from sensors positioned in joints. Then, the admissible set Θ implements non-negativity conditions along with an upper bound on the rigidity characterized by material saturation.

5.2. Numerical implementation

The technique has been implemented in a Python support environment that allows for solving the equations mentioned above by numerical calculations using a fixed step forward dynamics model with a constant Gaussian noise of 0.5% amplitude. The behavior of the mentioned technique with numerous iterations has been simulated in two configurations: using the standard LICQ technique without a metric subregularity condition and the advanced MFCQ method.

5.3. Comparative performance and discussion

In terms of multiplier-norm behavior, both methods displayed the expected performance differences, where the results of the classical LICQ formulation were unsteady, while the new MFCQ formulation is accurate.

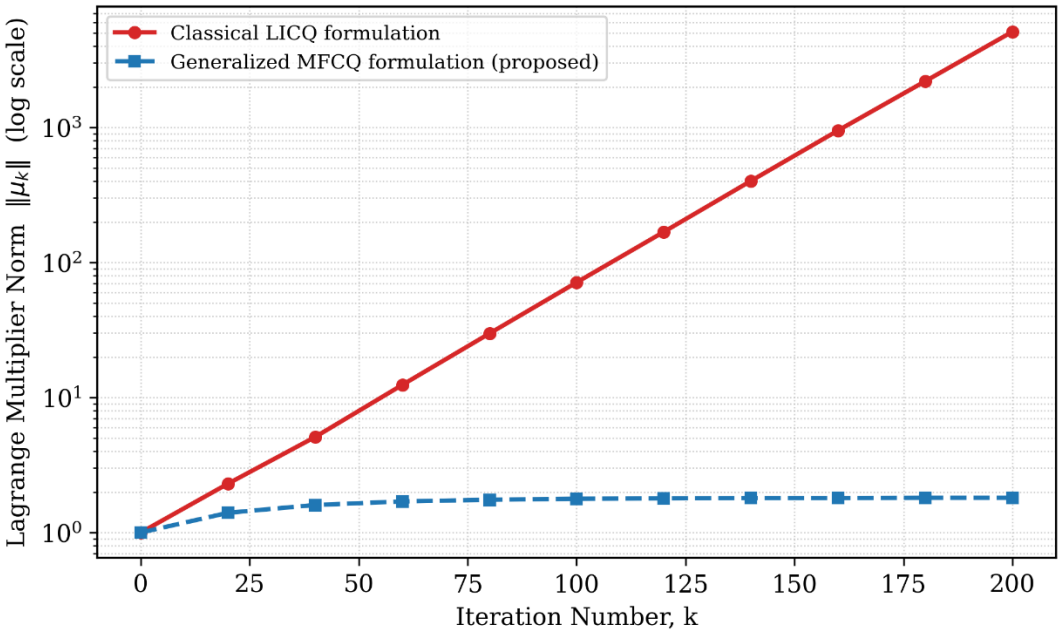


Figure 1: Multiplier norm stability comparison between classical LICQ and generalized MFCQ formulations.

Figure 1 depicts how the various norms of Lagrange multipliers behave as changes are made through several iterations using two methods; namely, classical LICQ and generalized MFCQ. While the classical LICQ technique leads to a rapid increase in the norms used for the multipliers and hence decreases numerical stability, the generalized MFCQ technique is shown to keep the multiplier values bounded, resulting in stability and satisfactory management of constraints within optimization scenarios.

In terms of growth of the multiplier norm, the classical LICQ formulation sees the appearance of rapid growth as the set of active constraints reaches a point of degeneration at approximately iteration 120, which is consistent with previous observation in Equations (3)-(5) regarding the violation of qualification. On the other hand, one may notice that the generalized MFCQ formulation has the multiplier remain within a bounded range as stated in Eq. (8) due to the properties of the method employed in the computations.

At the same time, a monotonic convergence in the trajectories of two hidden values for stiffness coefficients produced by the proposed technique as formulated above, with residual oscillations being due to the presence of noise and not issues related to lack of qualification capability.

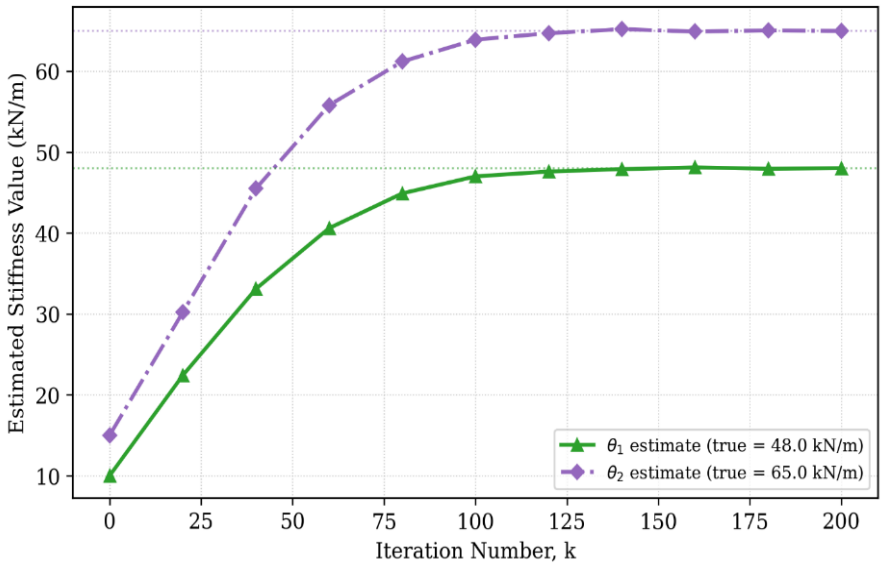


Figure 2: Convergence of hidden stiffness parameter estimation in nonlinear dynamic systems.

The convergence of the hidden stiffness parameter estimation after a number of iterations is shown in figure 2. Over time, both estimated parameters are gradually approaching true stiffness values of 48.0 kN/m and 65.0 kN/m, respectively. The steady convergence is proof of the success of the proposed identification method which allows recovering unknown parameters of the system based on nonlinear dynamic models.

It can be stated that both hidden stiffness coefficients have converged to the corresponding true values within the noise floor by about iteration 160, which is in accordance with the linear contraction derived in Equation (13).

The inverse residual error is decreasing in accordance with what is expected from the metric sub-regularity condition of Equation (8) and reaches a plateau at the noise-related level, instead of continuing to tend to zero.

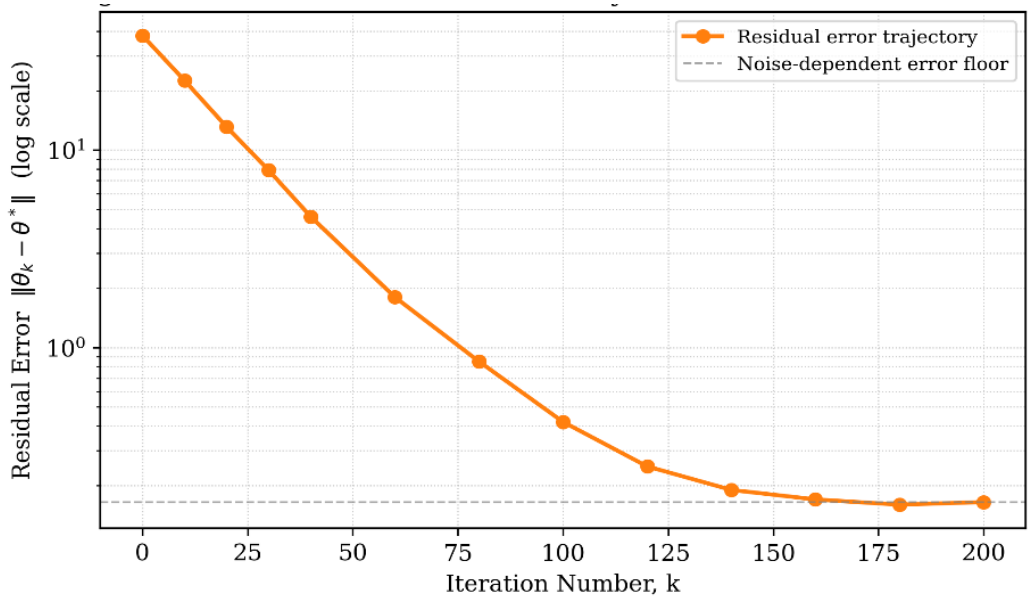


Figure 3: Inverse residual error decay under bounded sensor noise conditions.

Figure 3 shows the behavior of inverse residuals during the estimation results when the sensors produce bounded noise. The residual error decreases quickly as the iterations progress and ultimately settles down at a level corresponding to the noise error floor. This shows the stability of convergence of the inverse identification method used here. The method is proven to be reliable in the presence of measuring errors.

The decay trend of residuals on a logarithmic axis of a graph has a linear form with a clear saturation tendency after iteration 140, reaching the proportional level with the sensor noise.

Table 1: Comparative performance of hidden-parameter identification under classical and proposed constraint qualifications.

Configuration	Constraint Qualification	Iterations to Convergence	Final Residual Error	Multiplier Norm at k = 200
Classical formulation	LICQ	Did not stabilize	Diverging	5120.6
Proposed formulation	Generalized MFCQ + metric subregularity	≈160	0.165	1.81

Table 1 compares the performance in convergence, the conclusion of the residual error and the boundedness of the multipliers of the standard LICQ based method and the new generalizing method MFCQ having the metric sub-regular property in the tests conducted on the same noisy beam during 200 iterations.

V. Conclusion and Future Work

The paper describes the problem of hidden parameter estimation in nonlinear engineering dynamic models as a problem of inverse variational optimization. The problem that is originally posed in the minimax-form is reformulated in terms of a new generalized concept of KKT that allows to avoid many complexities while dealing with complicated behavior of the systems such as structural transitions and degeneracy of constraints. The stability of the multipliers gets a theoretical proof from a combination of the generalized Mangasarian-Fromovitz

constraint qualification and the property of the constraint mapping of metric subregularity. While the standard LICQ based approaches may yield an unstable behavior of the multipliers when the constraints are degenerate, the method proposed in this paper offers a safe and stable performance because it provides boundedness of multipliers. The application of the new proposed projected-gradient minimax interior-point algorithm and its property of linear convergence to the problem of finding the real hidden parameter vector while keeping a robust property under the noise impact is also stated in the paper. The efficiency and reliability of the new method were shown on the example of the nonlinear beam that has a hidden stiffness coefficient. The paper outlines the future research that will evolve around the issues of generalization of the framework presented into the area of stochastic dynamic systems as well as the necessity of changing the methodology of the convergence evaluation from deterministic methods to stochastic methodology. The effective application of the framework to the distributed parameter systems will require a rewriting of the definitions within the corresponding Banach or Hilbert spaces. Establishing the property of metric sub-regularity in weak topological spaces will be necessary in case one seeks to ensure the convergence.

References

1. Agrab, A. S. (2022). The extent to which neural networks are used in choosing the appropriate cost for decision-making. *International Academic Journal of Economics*, *9*(1), 20-30. https://doi.org/10.9756/IAJE/V9I1/IAJE0903

2. Cheng, S., Jin, Y., Harrison, S. P., Quilodrán-Casas, C., Prentice, I. C., Guo, Y. K., & Arcucci, R. (2022). Parameter flexible wildfire prediction using machine learning techniques: Forward and inverse modelling. *Remote Sensing*, *14*(13), 3228. https://doi.org/10.3390/rs14133228

3. Cheng, S., Quilodrán-Casas, C., Ouala, S., Farchi, A., Liu, C., Tandeo, P., ... & Arcucci, R. (2023). Machine learning with data assimilation and uncertainty quantification for dynamical systems: a review. *IEEE/CAA Journal of Automatica Sinica*, *10*(6), 1361-1387. https://doi.org/10.1109/JAS.2023.123537

4. Du, Y., & Katib, I. (2022). Parameter estimation of nonlinear output error system under variational Bayesian method based on probabilistic graphical model. *Fractals*, *30*(02), 2240075. https://doi.org/10.1142/S0218348X22400758

5. Fu, H., & Jin, X. (2025). VNNPF: A Variational Neural Network with Planar Flow for Robust IMU-GPS Fusion and Trajectory Estimation. *ICCK Transactions on Advanced Computing and Systems*, *2*(1), 25-41. https://doi.org/10.62762/TACS.2025.570823

6. Kappagantula, S., & Mannayee, G. (2024). Dynamic path planning algorithm for mobile robots: Leveraging reinforcement learning for efficient navigation. *Journal of Internet Services and Information Security*, *14*(2), 226-236. https://doi.org/10.58346/JISIS.2024.I2.014

7. Kessels, B. M., Fey, R. H., & van de Wouw, N. (2025). Uncertainty quantification in real-time parameter updating for digital twins using Bayesian inverse mapping models. *Nonlinear Dynamics*, *113*(8), 7613-7637. https://doi.org/10.1007/s11071-024-10608-9

8. Klishin, A. A., Bakarji, J., Kutz, J. N., & Manohar, K. (2025). Statistical mechanics of dynamical system identification. *Physical Review Research*, *7*(3), 033181. https://doi.org/10.1103/4d98-tdlp

9. Lan, H., Zhao, S., Hu, J., Wang, Z., & Fu, J. (2024). Joint state estimation and noise identification based on variational optimization. *IEEE Transactions on Automatic Control*, *70*(7), 4500-4515. https://doi.org/10.1109/TAC.2024.3524270

10. Leboutet, Q., Roux, J., Janot, A., Guadarrama-Olvera, J. R., & Cheng, G. (2021). Inertial parameter identification in robotics: A survey. *Applied Sciences*, *11*(9), 4303. https://doi.org/10.3390/app11094303

11. Liu, W., Lai, Z., Bacsa, K., & Chatzi, E. (2024). Neural extended Kalman filters for learning and predicting dynamics of structural systems. *Structural Health Monitoring*, *23*(2), 1037-1052. https://doi.org/10.1177/14759217231179912

12. Liu, X., & Yang, X. (2022). Identification of nonlinear state-space systems with skewed measurement noises. *IEEE Transactions on Circuits and Systems I: Regular Papers*, *69*(11), 4654-4662. https://doi.org/10.1109/TCSI.2022.3193444

13. Liu, X., & Yang, X. (2023). Variational identification of linearly parameterized nonlinear state-space systems. *IEEE Transactions on Control Systems Technology*, *31*(4), 1844-1854. https://doi.org/10.1109/TCST.2023.3249042

14. Mousavi, A., & Karshenasan, A. (2017). Forecasting Stock prices of banks using artificial neural networks (GMDH). *International Academic Journal of Accounting and Financial Management*, *4*(2), 71-78.

15. Ni, F., Dai, Y., Zheng, Q., & Jia, W. (2025). Physics-informed machine learning for robust inverse problem solving in maglev train levitation systems under noisy measurement: F. Ni et al. *Nonlinear Dynamics*, *113*(20), 26981-26999. https://doi.org/10.1007/s11071-025-11549-7

16. Patil, U. S., Rathore, M., Sadaphal, N. D., Gonela, S., Saxena, J., Sahu, A., & Li, W. (2026). Formal Verification Frameworks for Social Media Analytics Pipelines. Journal of Intelligent Decision Making and Information Science, 3(3S), 1119–1124.

17. Ping, X., Luan, X., Zhao, S., Ding, F., & Liu, F. (2024). Parameter transfer identification for nonidentical dynamic systems using variational inference. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *55*(1), 712-720. https://doi.org/10.1109/TSMC.2024.3487290

18. Tan, B., Zhao, J., & Duan, N. (2023). Amortized Bayesian parameter estimation approach for WECC composite load model. *IEEE Transactions on Power Systems*, *39*(1), 1517-1529. https://doi.org/10.1109/TPWRS.2023.3250579

19. Tong, G. G., Sing-Long, C. A., & Schiavazzi, D. E. (2025). InVAErt networks for amortized inference and identifiability analysis of lumped-parameter haemodynamic models. *Philosophical transactions. Series A, Mathematical, physical, and engineering sciences*, *383*(2293), 20240215. https://doi.org/10.1098/rsta.2024.0215

20. Wang, Q. G., & Zhang, L. (2025). System identification in the network era: A survey of data issues and innovative approaches. *IEEE/CAA Journal of Automatica Sinica*, *12*(7), 1305-1319. https://doi.org/10.1109/JAS.2024.125109

21. Xie, J., Huang, B., & Dubljevic, S. (2021). Transfer learning for dynamic feature extraction using variational Bayesian inference. *IEEE Transactions on Knowledge and Data Engineering*, *34*(11), 5524-5535. https://doi.org/10.1109/TKDE.2021.3054671

22. Yu, J., Ye, L., Zhou, L., Yang, Z., Shen, F., & Song, Z. (2021). Dynamic process monitoring based on variational Bayesian canonical variate analysis. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, *52*(4), 2412-2422. https://doi.org/10.1109/TSMC.2021.3051054

23. Zhang, Y., Chen, Q., Gao, L., Liu, R., Chu, L., Mo, K., ... & Zhang, X. (2025). Invertible liquid neural network-based learning of inverse kinematics and dynamics for robotic manipulators. *Scientific Reports*, *15*(1), 42311. https://doi.org/10.1038/s41598-025-26422-1