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Mixed-integer programming formulations for resource scheduling and operational planning in industrial networks

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Abstract:

Problems related to mixed-integer programming with respect to resource scheduling and operational planning need mathematical models that can incorporate both discreteness, continuity and conflicting objective functions. Traditional methods of mixed-integer programming are effective in expressing feasibility constraints and combinatorial structures of the decision process; but they do not have explicit ways of balancing worst-case objective behavior when the objective function is multi-objective. In this research, a framework of mixed-integer minimax optimization that utilizes minimax robustness with mixed-integer decision process modeling is introduced as an approach for balanced optimization in a constrained environment. The proposed formulation is an amalgam of continuous resource variables, discrete scheduling variables, logic constraints, and multi-objective functions expressed as a minimax objective. Existence of optimal solutions and relaxations-based bounds that enable the implementation of the branch-and-bound optimization method are analyzed theoretically. The computational model is tested numerically by using 50 decision variables, 120 constraints, and different optimization cases. The proposed Minimax-MIP model resulted in an objective function value of 118.7 with a minimized optimality gap of 1.9% compared with the other methods, including

linear programming relaxation, genetic optimization, and traditional MIP optimization methods. The convergence and sensitivity analysis show the efficiency of the model.

Keywords: Mixed-integer programming, minimax optimization, resource scheduling, operational planning, mathematical optimization, branch-and-bound, multi-objective optimization.

1. Introduction

Planning and scheduling problems are often defined by complicated decision-making scenarios consisting of discrete schedule decisions, continuous resource allocations, and dependent constraints. Mixed-Integer Programming (MIP) framework can be used in modeling such types of problems using combinations of integer variables for selection, assignment, and sequencing decisions with continuous variables for resource use and assessment. Recent studies that proposed formulations in the area of MIPs showed that the proposed formulations were effective in modeling large-scale scheduling and planning problems through enhanced constraint representation and better accuracy in solving under realistic operational conditions (Bamoumen et al., 2025). Prior research into formulations of network scheduling problems using MIPs showed that MIPs are capable of modeling complicated relationships between processes, resources, and operations while keeping computational efficiency (Stebel et al., 2012).

Mixed-Integer approaches have found wide applications in the areas of production scheduling, network optimization, maintenance planning, and capacity management problems. Formulations in the field of MIP helped to make effective decisions using logic constraints, resource constraints, and time constraints in an optimization model (Sundaramoorthy & Maravelias, 2011). The application of optimization techniques in resource allocation and scheduling problems with several operational elements is another example of MIP's effectiveness in handling discrete-continuous decision problems (Akbaripour et al., 2018). Decision support methods using computational intelligence pointed out the necessity of mathematical optimization models in making effective resource selection and cost-aware planning processes. Dynamic multi-objective evolutionary VM placement algorithms can be used for optimizing resource management in cloud data centers by taking into consideration different objectives (Torre et al., 2020).

Despite the powerful modeling capabilities offered by conventional MIP models, most current methods either address single-objective optimization or consider weighted objective formulations. Conflicting objectives, including cost reduction, performance improvement, and system reliability, require optimization algorithms that can find a proper balance between competing criteria. The study of integrated production and maintenance scheduling has shown the necessity of constructing a complete mathematical model taking into account all operational constraints (Chansombat et al., 2019). Hybrid optimization algorithms, which combine mathematical programming with heuristics and are applied to solve multi-objective scheduling problems, were also considered; yet, they concentrate mainly on computation efficiency and not the explicit worst-case objective control (Şahin & Kellegöz, 2019). The AgriSynth system utilizes the concept of synthetic data generation along with deep ensemble regression for predicting agricultural crop yield (Sahu et al., 2026).

Minimax optimization is an efficient mathematical tool that minimizes the maximal deviations among several objectives and is therefore suitable for decision-making problems under uncertainty and conflicting criteria. Available mixed-integer formulations for scheduling and planning problems provide the basis for the integration of the minimax optimization techniques and decision-making structure (Artigues et al., 2014). Recent advances in multi-objective optimization have also shown the necessity of constructing a balance-oriented approach to obtain better trade-off solutions between conflicting criteria (Mokhtari et al., 2025). This research aims at developing a mixed-integer minimax optimization algorithm, which includes both integer feasibility and worst-case objective balance (Tonelli et al., 2013). The algorithm provides the theoretical properties of the existence of the optimal solutions and relaxation-based bounds as well as a computational approach for application in resource scheduling and planning problems (Garcia-Sabater et al., 2012). The developed approach expands the applicability of conventional MIP models due to the integration of minimax robustness (Tavaghof-Gigloo et al., 2016; Kadhim & Shani, 2023).

Contribution of the current paper: (i) formulation of the minimax mixed-integer optimization model; (ii) mathematical definition of the feasibility and optimality conditions; (iii) branch and bound methodology with the convergence proof; and (iv) validation via numerical examples.

This paper is structured in the following way. Section 1 explains the research background, motivations, and contributions of the mixed integer minimax optimization approach. Section 2 gives mathematical preliminaries and problem formulations. The methodology of the Mixed Integer Minimax Optimization Problem is elaborated on in Section 3. Section 4 presents the theoretical and optimality analyses. Computation method and complexity analysis are presented in Section 5. The results of numerical experiments and their analysis are given in Section 6. Section 7 is the conclusion.

2. Mathematical preliminaries and problem definition

2.1 Mixed-integer decision space

The decision domain consists of a set of resource allocation variables that are continuous together with scheduling decision variables that are discrete. The continuous part reflects quantities that can be divided, whereas the discrete part represents allocation, sequencing, and on/off decisions. The decision domain is formulated as follows (Equation 1).

$$X = \{(x, y): x \in R^n, y \in Z^m\} \quad (1)$$

2.2 Minimax optimization framework

The minimax paradigm involves making decisions such that the maximum or worst value of several objective functions is minimized. The reason why this paradigm is robust is that it clearly defines bounds on the least favorable outcome instead of the average outcome and resolves conflicts between objectives by converging them to the same value (Equation 2).

$$\min_{x,y} \max_{i=1,\dots,m} f_i(x, y) \quad (2)$$

2.3 Problem statement

Introducing the auxiliary scalar epigraph variable transforms the minimax problem into a typical mixed-integer programming problem in which the worst-case objective function value is minimized under the constraint that all individual objectives as well as the resource and logical constraints must be less than the scalar value (Equations 3 – 6).

$$\min_{x,y} \lambda \quad (3)$$

Subject to:

$$f_i(x, y) \leq \lambda \quad (4)$$
$$Ax + By \leq b \quad (5)$$
$$y_i \in \{0,1\} \quad (6)$$

3. Proposed mixed-integer minimax formulation

This part will introduce the main contribution to this work, namely the unifying mixed-integer minimax problem formulation that considers all the aspects described before (continuous resource variables, scheduling discrete variables, coupling constraints, multi-objective framework, and all reduced to one worst-case objective).

3.1 Variable construction

Continuous variables represent divisible resources like processing time, material flow or capacity usage, whereas discrete integer variables represent non-divisible decisions like machine allocation or sequencing tasks x_i and y_i.

3.2 Multi-objective mathematical model

The method combines k different performance measures, including cost, tardiness, and imbalance of resources, into a vector-valued objective function that is then transformed into a scalar minimax objective function (Equations 7&8).

$$F(x, y) = \big(f_1(x, y), f_2(x, y), \dots, f_k(x, y)\big) \quad (7)$$

$$\min_{x,y} \max F(x, y) \quad (8)$$

3.3 Constraint model

Feasibility constraints fall into three categories (Equations 9-11): an upper bound on aggregate resource capacity, a logical big-M relationship that triggers continuous variables only if the corresponding discrete variable choice is made, and completeness of the schedule, where each task receives precisely one assignment.

$$\sum_i a_i x_i \leq C \quad (9)$$

$$x_i \leq My_i \quad (10)$$

$$\sum_t y_{it} = 1 \quad (11)$$

3.4 Conceptual mathematical framework diagram

Figure 1 depicts the conceptual framework of the suggested mixed integer minimax optimization model, which reflects the gradual process of transformation of the decision-making process. The framework commences with the delineation of the decision space, which is followed by the transformation of continuous and discrete variables into one consistent picture. The constraint operators help formulate the feasibility criteria, whereas the multi-objective functions reflect the conflicts within the criteria for optimization.

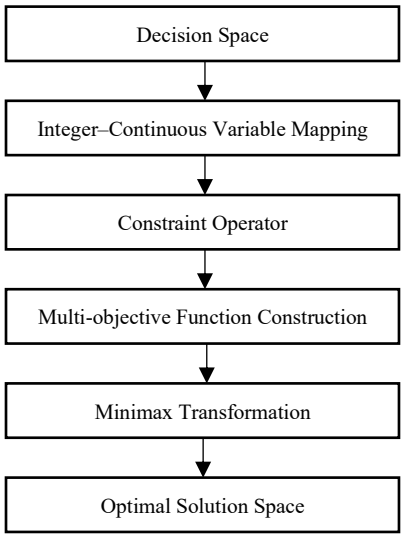


Figure 1: Mixed-integer minimax optimization framework.

4. Theoretical analysis

Theorem 1: Existence of optimal solution

Statement: For a bounded feasible mixed-integer decision space and continuous objective functions, the proposed minimax formulation admits an optimal solution.

Assumption: The feasible set X is nonempty, closed, and bounded, and each f_i is continuous in x for every fixed y ∈ Z^m .

Proof: Due to boundedness of y in the finite feasible integer set, the problem breaks down into finitely many continuous problems, all over compact sets. Based on Weierstrass' extreme value theorem, the continuous function $max_i f_{i(x,y)}$ achieves its minimum in each compact subproblem, and comparison among the finite subproblem minima gives the optimal solution.

Conclusion: An optimal solution (x^*, y^*) of the mixed-integer minimax problem exists based on the above boundedness and continuity assumptions.

Theorem 2: Optimality bound of relaxed formulation

Statement: The continuous relaxation formulation, which involves relaxing the integer variables y to the interval of real numbers, serves as a lower bound to the original minimax mixed-integer problem.

Proof: Relaxation of $y \in Z^m$ to $y \in [0,1]^m$ expands the feasible region, because every feasible mixed-integer solution remains feasible in the relaxed formulation and some new feasible solutions also arise. This implies that minimization over an expanded feasible region will not give rise to a larger optimal value, which leads to an upper bound for the relaxed problem optimum (Equation 12).

$$F_R \leq F_M \quad (12)$$

where F_R denotes the optimal continuous relaxation value, and F_M denotes the optimal mixed-integer minimax value; this bound underlies the branch-and-bound pruning strategy used in the solution procedure.

5. Computational solution and complexity analysis

5.1 Solution procedure

The suggested algorithm employs a branch-and-bound strategy in which the relaxation problem is solved for LB, the region of feasibility is partitioned based on fractional integer variables, mixed-integer feasible solutions are used to update UB, and branches are fathomed whenever bound gaps close based on the tolerance rule (Equation 13).

$$UB - LB \leq \varepsilon \quad (13)$$

5.2 Complexity discussion

Solution difficulty increases with an increase in both the number of continuous variables and the number of binary variables because more continuous variables will make the size of the relaxation larger, and an increase in the number of binary variables will lead to an exponential expansion of the integer solution space. Tight relaxations and bound propagation significantly improve efficiency through pruning of branches.

6. Results and discussion

6.1 Experimental parameter setup

Evaluations of the suggested framework were made based on an operational planning problem set up to be of realistic scheduling complexity by having fifty total decision variables, twenty scheduling decision variables, and thirty continuous decision variables within a total constraint system of one hundred twenty constraints. The parameters used to evaluate the suggested mixed-integer minimax optimization framework are illustrated in table 1. The dimensions of the decision space that have been considered include the number of total decision variables, number of binary decision variables, number of continuous decision variables, constraints, optimization iterations, and convergence tolerance.

Table 1: Optimization configuration.

Parameter	Value
Decision variables	50
Binary variables	20
Continuous variables	30
Constraints	120
Iterations	500
Tolerance	10^{-6}

6.2 Comparative optimization results

Minimax-MIP approach is evaluated relative to LP relaxation, genetic optimization algorithm, and traditional MIP approach not involving minimax balance. The suggested approach provides the best results in terms of objective function values and optimality gaps with decent computational time proving the advantage of minimax approach combined with MIP solving. Comparison of different solution methodologies in terms of objective value, optimality gap, and computation time is provided in table 2. Comparison is done with LP relaxation, genetic algorithm optimization, traditional MIP, and the proposed Minimax-MIP approach. It has been demonstrated that better solution values, optimality gap reduction, and faster computation are possible using the proposed model.

Table 2: Performance comparison.

Method	Objective Value	Optimality Gap (%)	Runtime (s)
LP Relaxation	142.8	18.5	42
Genetic Optimization	131.6	10.3	185
Conventional MIP	125.4	5.8	96

Proposed Minimax-MIP	118.7	1.9	72

6.3 Convergence behavior

The pattern exhibited by the minimax objective function values along the line of convergence shows that there is an initial fast improvement in the value before stabilization toward the final converged value, meaning that the bound tightening algorithm works effectively during branching. Figure 2 shows the behavior of the objective value of minimax over the process of optimization. This value declines continuously starting from the initial value to reach the optimum one showing efficient solution refining and optimization stabilization as well as the accomplishment of the minimization objective value.

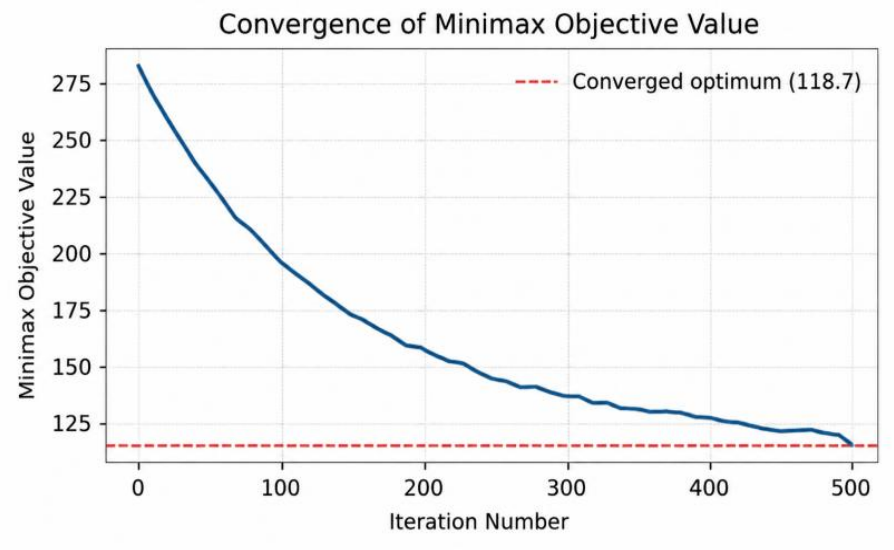


Figure 2: Convergence of minimax objective value.

6.4 Sensitivity analysis

An increase in the number of decision variables increases the objective function value and computation time, due to the combinatorial increase of the integer search space. However, the increase in computation time is not too big compared to the growth of problem size, which means that the approach of using relaxation-based branching is sufficiently contained. Table 3 shows the sensitivity analysis of the mixed-integer minimax model formulation based on the changing decision variable sizes. In this table, the effect of the change in the problem dimension on the objective function value and computation time is evaluated.

Table 3: Sensitivity analysis.

Variable Size	Objective Change	Runtime Increase
50	Baseline	72s
100	+8.4%	115s
250	+18.7%	210s

6.5 Objective trade-off visualization

Placing objective 1 versus objective 2 for individually optimized points follows the Pareto trade-off curve, whereas the minimax solution point achieves a balance on the curve without the bias towards either objective that would have resulted from the individual optimization. Figure 3 depicts the trade-off between conflicting objectives by means of the Pareto curve. The minimax solution point is the balance optimization point where both objectives are taken into account, proving the ability of the proposed formulation to perform well under competitive performance criteria.

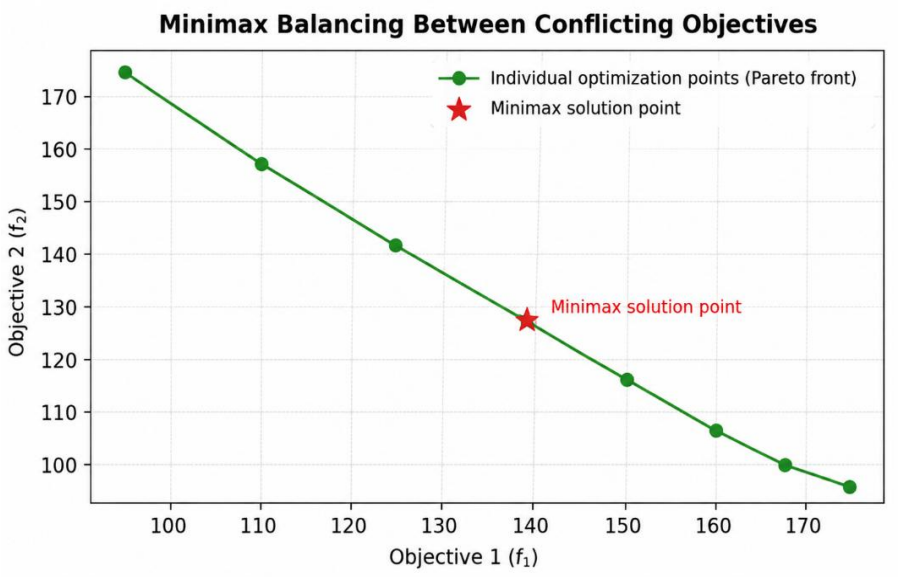


Figure 3: Minimax balancing between conflicting objectives.

7. Conclusion and future scope

The proposed study suggested a new framework of mixed-integer minimax optimization to solve resource scheduling and operational planning problems with discrete, continuous decisions and multiple conflicting objectives. The formulated mathematical optimization model combined the mixed-integer programming techniques with the concepts of minimax optimization to ensure balance in the decision-making process under constraints. Moreover, the proposed model proved the existence of optimal solutions in the decision space and provided the relaxation-based solution quality assessment via deriving the corresponding bounds. The novel branch-and-bound optimization framework was designed to search the mixed-integer solution space in an efficient manner while ensuring the convergence of the algorithm and its optimality. The computational investigation proved the effectiveness of the suggested Minimax-MIP approach by attaining better objective values, smaller optimality gaps, and competitive computational performance compared with the linear programming relaxation, genetic optimization, and classic MIP techniques. The obtained results demonstrate the ability of the suggested approach to provide robust and reliable optimization solutions in the complex decision environment.

Further research will be devoted to the development of the proposed framework toward advanced mathematical optimization cases, including the aspects of uncertainty, scalability, and dynamics of the decision-making process. The stochastic mixed-integer minimax optimization models will be developed to account for the uncertainties in the operational parameters. The distributed minimax optimization strategies will be considered to enable efficient computation in the context of large-scale networked systems with distributed decision variables. Additionally, the robust optimization techniques taking into account uncertain coefficients, dynamic constraints, and variable perturbations will be explored to improve adaptability in real-world situations. Finally, the extension of the suggested framework toward infinite-dimensional optimization models will allow one to conduct continuous-time operational planning and decision-making processes.

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