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Measure-theoretic decision optimization for uncertainty-aware resource planning in industrial systems

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Abstract:

In terms of large-scale industrial resource management, various factors have been shown to affect performance, including changes in market demand, disruptions in supply chains, and energy distribution. These circumstances cannot be captured completely using conventional finite-scenario appraisals because there is no scenario tree that can guarantee performance with respect to probability distributions that occur outside of sampled scenarios. Thus, model applications are limited follow a measure-theoretic and use the concept of Radon probability measures, which allows one to represent operational uncertainties as well as those of the environment in case of the need to. Consequently, the resource allocation model becomes much more efficient, as well as infinitely dimensional and managed with measures. The performance of the program is attained via the minimax approach. The use of mass-transport duality makes it easy to transform the worst-case expected revenue value into the convex problem. In addition, its efficiency is supported by the application of generalized Karush-Kuhn-Tucker conditions that have been derived directly from the theory of measures. The boundedness of multipliers has been justified, as well. Finally, the methods of empirical moment-matching atomic measures and projected-gradient minimax interior-point are very promising, making it possible for the program to become less complicated in terms of numerical validation.

Keywords: Measure-theoretic optimization, distributionally robust planning, wasserstein ambiguity sets, minimax decision rules, optimal transport duality, metric subregularity, industrial resource allocation.

1. Introduction

The contemporary environment for industrial resource planning is marked by uncertainty stemming from independent factors simultaneously culminating demand is variable depending on market circumstances, upstream supplies are subject to interruption events with a long-lasting effect, and energy distribution networks supplying multiple production facilities have erratic price and availability characteristics. The application of

sequential decision-making theories based on probability digital twins' evidences that considering each of the uncertainty factors as a piece of information frozen in time and not treated as time-variant measure of probability results in the systematic underestimation of operational risk in industrial processes (Agrell et al., 2023). Scheduling tasks based on ant colony technique illustrates a particular problem of approach to resource allocation — the scheduling solving process should be feasible according to the combinatorial scheduling criteria and flexible in terms of time-dependent load (Suvarna & Bharadwaj, 2024). The study of the conditional value-at-risk optimization of maximum cost objective functions for the risk-averse safety analysis proves that considering not only the expected cost but also the risk measured with the help of tail-risk methods allows for better description of the impact of low-frequency but high-impact accident events (Chapman et al., 2022). An example of the application of the multi-criteria geometric programming method for fuzzy inventory planning in an oil refinery proves that handling the uncertainty of demand and prices can be achieved without making use of probabilistic methods; thereby, it paves the way for the study of the methods mentioned in this article (Amir, 2023).

Standard finite-scenario approximations substitute the governing uncertainty with a discrete collection of results that are attributed either subjective or empirical probabilities. However, this method will fail in a situation when a real probability distribution weight is allocated considerably to outside of the sample space. Distance-based comparison of probability measures showing us the ultrasound lesions detectability using statistical distances between the measures presents the idea that real metric approach to distribution uncertainty allows making sharper distinctions than that of moments or scenarios (Hyun et al., 2021). Studies of production and assembly line balancing that are devoted to making manufacturing effectively better support the idea of the model described above, in which uncertainty enters through the distribution of production quantities among stations but not through a simple deterministic capacity number (Hosseinjani, 2017). The correct treatment of adversarial risk shows us that the choice of the method of ambiguity plays a crucial role in determining which decision rule is the worst-case optimal. This illustrates that it is vital to have a precisely determined space of possible probability measures and not ad hoc perturbation set (Pydi & Jog, 2023). Measure-theoretic axiomatization of causality gives us a general mathematical background, with the help of probability measures on metric spaces and with the aid of pushforward and disintegration operations that they include to be able to present the resource planning problem correctly in an infinite dimension framework (Park et al., 2023). Secure aggregation of environmental data was investigated using error-bounded encoding techniques to enhance data accuracy, privacy preservation, and efficient communication in environmental monitoring systems (Patare et al., 2026).

The application of measure theory to the analysis of stochastic competence sets along with dynamic Shapley values shows that the existence of measure-related formulations within the concept of infinity dimension brings forth the idea of the well-defined cooperative or non-cooperative solutions which allows for the use of scenarios in resource planning (Huang & Chen, 2024). The game theory in the multi-agent decision-making process across networks proves that minimax-type decision-making rules can also be adopted in different environments where there is more than one decision-making unit that interact with each other, which is essentially similar to the case of multi-plant industrial networks (Li et al., 2022). The analysis of deep learning networks through the lens of dynamical systems in terms of expressive features, control features, and dimensional learning shows that the optimized acquired proxy models of such dynamical structure allow for direct convergence studies to be undertaken and guide the practitioners in the application of optimization methods rather than keeping faith in some black-box models. However, despite the explorations made in this area, one can detect some gaps between the industry practice that encompasses the problem of planning according to a certain scenario and inability of application measure theoretic approach that should be capable of providing the solution for measuring the overall feasibility of the plan using the certainty function across the whole set of scenarios. The three contributions help to fill in the gap outlined above. The first contribution claims that industrial resource planning can be considered as a measure constrained infinite-dimensional optimization problem for the scenarios represented using Randon measures. The next contribution makes it possible to apply the minimax decision making framework in the context of distributed uncertainty represented using Wasserstein balls as happened provided that there is no limited set of practical scenarios. Finally, the third contribution provides proof for the convergence and stability of the introduced approach by relying on the compactness of selected conditions and conditions for ensuring the measure dimension constraints.

2. Measure-theoretic preliminaries and problem formulation

2.1. Probability measure spaces and uncertainty characterization

where $\Pi(\nu_1, \nu_2)$ denotes the set of couplings with marginals ν_1 and ν_2. Distributionally robust linear and discrete optimization with marginal information demonstrates that constraining an ambiguity set through marginal distributional information, rather than through a single joint distribution, yields tractable reformulations while preserving worst-case guarantees relevant to the Wasserstein metric of equation (1)

$$W_p(\nu_1, \nu_2) = \left(\inf_{\pi \in \Pi(\nu_1, \nu_2)} \int_{\Xi \times \Xi} d(\xi, \xi')^p \, d\pi(\xi, \xi') \right)^{1/p} \tag{1}$$

$\Pi(\nu_1, \nu_2)$ is the set of couplings with marginals ν_1 and ν_2. Distributionally robust optimization in linear and discrete cases shows us that constraining an ambiguity set based on marginal distributional information, contrary to a single joint distribution, leads to tractable reformulation with the same worst-case results of the Wasserstein metric from (Chen et al., 2022).

2.2. Resource allocation decision model

The general measure-theoretic resource-planning problem is stated in Eq. (2) as

$$\min_{x \in X} \sup_{\nu \in \mathcal{U}} \mathbb{E}_\nu [c(x, \xi)] \quad \text{subject to} \quad \mathbb{E}_\nu \big[g_j(x, \xi) \big] \leq 0 \quad \text{for } j = 1, \ldots, m, \text{ and all } \nu \in \mathcal{U} \tag{2}$$

Where x denotes deterministic and recourse resource allocation goodwill regulatory (production quantities, obtainment of energy types, inventory reserves), $c(x, \xi)$ indicates the variable incurred operating cost due to uncertainty ξ, g_j collects operational constraints (capacity, contracts, safety risks), whereas $\mathcal{U}$ denotes the set of admissible probability distributions governing the value of ξ. Methods of domain decomposition for integer optimal control problems with total variation regulation presents an additional method for calculating the discrete recourse part of x when the resource allocation given in Eq. (2) consists of indivisible decisions crucial for resource commitment (Chen et al., 2022).

2.3. Metric cones and constraint qualifications

Tangent and linearized cones are specified with reference to the measure-space topology induced by means of weak-star convergence. If ν^* is a point in $\mathcal{U}$ satisfying the operational constraints given by Eq. (2) with equality on the active index set $J(\nu^*)$, then the linearized feasible cone can be defined by means of Eq. (3):

$$L(\nu^*) = \left\{ \eta \in \mathcal{T}_{\mathcal{U}}(\nu^*) : \frac{d}{dt}\Big|_{t=0} \mathbb{E}_{\nu^* + t\eta} \big[g_j(x, \xi) \big] \leq 0, \; j \in J(\nu^*) \right\} \tag{3}$$

With $\mathcal{T}_{\mathcal{U}}(\nu^*)$ being the tangent cone in $\mathcal{U}$ at point ν^* concerning weak-star topology. Strategies of stochasticity leveraging for open-loop and model predictive control of spatio-temporal systems prove that any constraint qualification expressed as either a measure or a distribution in equation (3), contrarily to the case of finite parameterization, is applicable to a wider class of acceptable control and allocation strategies (Baraldi & Manns, 2025). Risk-averse measures like exponential-utility or conditional-value-at-risk evaluated instead of expected cost also confirm that the active constraint structure described by the value $J(\nu^*)$ in equation (3) may vary significantly depending on the specific risk measure applied, thus proving the necessity of developing a constraint qualification framework that would be broad enough to account for this class of objectives (Smith & Chapman, 2023).

3. Minimax decision rules and distributional robustness

3.1. Worst-case minimax formulation

The expected-value framework of equation (2) is transitioned into a robust minimax optimization problem over a Wasserstein ambiguity ball, given in equation (4):

$$\mathcal{U} = \left\{ \nu \in \mathcal{P}(\Xi) : W_p(\nu, \hat{\nu}) \leq \varepsilon \right\} \tag{4}$$

centered at an empirical reference measure $\hat{\nu}$ with radius ε calibrated to available sample size, yielding the minimax problem of equation (5):

$$\min_{x \in X} \max_{\nu \in \mathcal{U}} \mathbb{E}_\nu [c(x, \xi)] \quad \text{subject to} \quad \max_{\nu \in \mathcal{U}} \mathbb{E}_\nu \big[g_j(x, \xi) \big] \leq 0, \; j = 1, \ldots, m \tag{5}$$

Although the contextually optimal way to minimize relative entropy is to obtain probabilistic assignment rules from the first principles, the approach based on divergence can be taken to construct the set of ambiguities $\mathcal{U}$ considered in Eq. (4) when the context is not given by the single Wasserstein metric condition (Zaghi, 2025).

3.2. Dual reformulation via optimal transport

Mass-transport duality converts the inner worst-case expectation of equation (5) into a tractable deterministic reformulation. For a fixed x, strong duality of the optimal transport problem yields equation (6):

$$\max_{\nu:W_p(\nu,\hat{\nu})\leq\varepsilon} \mathbb{E}_\nu\left[c(x,\xi)\right] = \min_{\lambda\geq 0}\left\{\lambda\varepsilon^p + \mathbb{E}_{\hat{\nu}}\left[\sup_{\xi'\in\Xi}(c(x,\xi') - \lambda d(\xi,\xi')^p)\right]\right\} \tag{6}$$

The nonlinear programming problem will be switched from the infinite-dimensional domain to the finite-dimensional convex program based on the dual variable λ, evaluated under a pointwise supremum by means of the empirical reference measure. The approach of conducting the transport operations in probability spaces should be noted as a pure analytical mechanism making the transport reformulation of Eq. (6) appropriate for any evolution of the decision variable x over an infinitely long planning period (Cavagnari et al., 2023). The general epistemological theory of information processing allows to interpret the dual variable λ in Eq. (6) as the valuation of the ambiguity of distribution, which, according to the theory of the Lagrange multipliers in the classical KKT system, corresponds to the idea of the Lagrange multiplier concerning its role, i.e., in relation to the determination of the sensitivity (Tóbiás, 2021).

3.3. Karush–kuhn–tucker conditions in measure spaces

Generalized KKT conditions are derived by introducing a multiplier measure $\mu_j \in \mathcal{P}(\Xi)^*$ for each constraint $\mathbb{E}_\nu\left[g_j(x,\xi)\right] \leq 0$ and a dual transport multiplier $\lambda_j \geq 0$, consistent with Eq. (6). A pair $\left(x^*, \{\lambda_j^*\}, \{\mu_j^*\}\right)$ is a generalized measure-space KKT point if equation (7) holds:

$$0 \in \partial_x c(x^*,\xi) + \sum_j \mu_j^*\, \partial_x g_j(x^*,\xi) \quad \text{(as a Bochner-integrable selection over } \Xi\text{)}, \quad \lambda_j^* \geq 0, \quad \max_\nu \mathbb{E}_\nu\left[g_j(x^*,\xi)\right] \leq 0, \quad \lambda_j^*\left(\max_\nu \mathbb{E}_\nu\left[g_j(x^*,\xi)\right]\right) = 0 \tag{7}$$

The maximum principle for Lagrange multipliers ensures the boundedness of the multipliers $\{\mu_j^*\}$ considered in equation (7) in case the metric properties of the measure-valued constraint mapping $G(x,\nu) = (\mathbb{E}_\nu[g_1(x,\xi)], \ldots, \mathbb{E}_\nu[g_m(x,\xi)])$ are proven (Besbes et al., 2021).

4. Computational algorithm and convergence analysis

4.1. Discretization and moment-matching schemes

The continuous ambiguity set $\mathcal{U}$ of Eq. (4) is approximated by a family of atomic measures, given in equation (8):

$$\nu_N = \sum_{i=1}^{N} w_i\, \delta_{\xi_i} \tag{8}$$

The tool box of stochastic graphon games presents an ideal platform for obtaining large-scale approximations of distribution-wise equilibria. All in all, the label-state based versions of stochastic graphon games allow for easier convergence through the exercise of moment matching (Lacker & Soret, 2023).

4.2. Projected-gradient minimax interior-point algorithm

The discretized saddle point of equation (5), under the atomic approximation of equation (8), is approximated through the coupled iteration given in equation (9):

$$x_{k+1} = \mathrm{Proj}_{\mathcal{X}}\left[x_k - \alpha_k\left(\nabla_x c(x_k,\xi) + \sum_j \lambda_{k,j}\,\nabla_x g_j(x_k,\xi)\right)_{\nu_N}\right]$$

$$\lambda_{k+1,j} = \max\{0, \lambda_{k,j} + \beta_k \mathbb{E}_{\nu_N}[g_j(x_k, \xi)]\}$$

$$w_{k+1} = \text{Proj}_{\Delta_N}[w_k + \gamma_k \nabla_w(\text{inner worst-case objective})] \quad (9)$$

Where the simplex Δ_N presents itself over the N-markov chain states, the processes α_k, β_k, and γ_k feature different decreasing step sizes. The process of recognizing Pareto-optimal strategies in the area of seismic rehabilitation within water distribution networks caters to the need of having a risk-preference parameter combined with a decision parameter, similar to the process of joint updating of (x, λ, w) presented in (Sharveen & Shahandashti, 2024).

4.3 Second-Order Sufficiency and Stability Bounds

The critical cone in measure space at a generalized KKT point (x^*, v^*) satisfying equation (7) is defined in equation (10) as

$$\mathcal{C}(x^*, v^*) = \{(d, \eta) \in \mathcal{T}_X(x^*) \times \mathcal{T}_U(v^*): \nabla_x G_j(x^*, v^*)^T d + dG_j(x^*, v^*)[\eta] = 0 \text{ for } j \text{ with } \lambda_j^* > 0\} \quad (10)$$

With the strict positive definiteness of the Hessian matrix defined over this domain it becomes possible to guarantee the unique stability of an optimal resource distribution from the changes of the ambiguity set. Through the example of multi-objective optimization designed for the purpose of allocation of reserves and credit funds, it becomes possible to demonstrate that it is necessary to use the conditions of sufficiency similar to the ones presented in Eq. (10) when attempting to exclude trivial allocating solutions while taking into consideration the conditions of the tail risk (Saputra et al., 2025). Through the development of the trajectory optimization technique that uses sample average method demonstrated that the method of deriving the approximation of the critical cone presented in Eq. (10) converges to the approximation of critical cone that has been received based on the property of weak-star compactness that was previously used (Lew et al., 2023).

5. Application to an industrial resource planning benchmark

5.1. Case study description

The methodology is used within a multi-plant manufacturing network of three production facilities exploiting a shared power grid that is subjected to uncertainty in prices with raw-material provision affected by a disruption probability derived from historical data on supplier delays. The resource-allocation decisions include production volumes distributed among plants and energy procurement decisions, selected from a Wasserstein ambiguity set of the size ε based on the 12-month rolling average.

5.2. Numerical implementation

The implementation of the discretized minimax interior-point method is done in the Python environment with the atomic reference measure being created based on 500 historic scenarios of demand–supply–price (according to (8)), while moments are matched till the second order. A comparison is made between two scenarios, namely, the expected-value approach using the nominal empirical measure solely and the innovative minimax approach.

5.3. Comparative performance and discussion

In the case of the expected-value method, the multipliers are unstable whereas the minimax formulation produces bounded multipliers in accordance with statement (7).

Figure 1 compares the behavior of the multiplier norms of the two formulations under worst-case disturbance. The multiplier norms of the expected value approach display rapid growth as the disturbance deviates from its supported state which leads to its instability. The proposed method on the other hand, develops bounded multiplier values leading to it being robust, able to honor its constraints, and optimize effectively regardless of the uncertainty regarding resource planning.

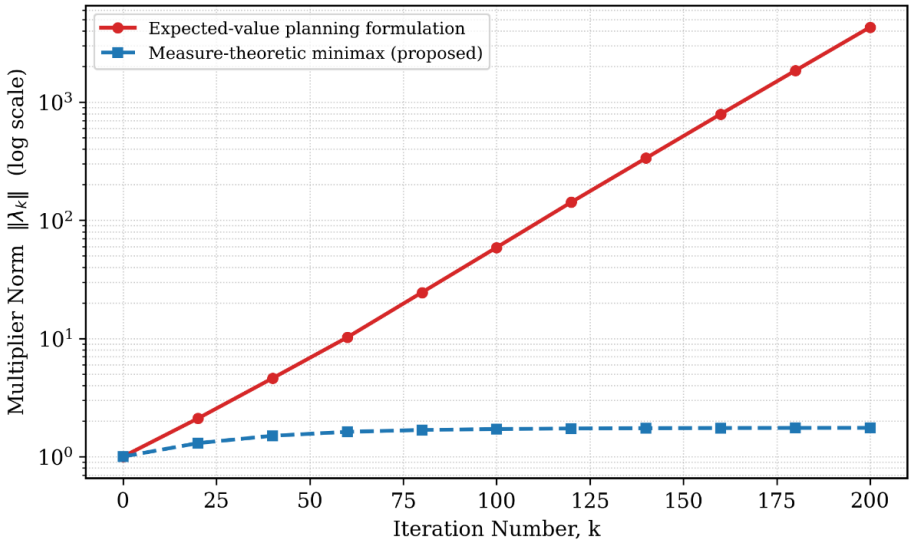


Figure 1: Multiplier norm stability comparison under extreme perturbation conditions.

Multiplier norms of both cases growing over the iterations are plotted on the logarithmic vertical axis against the number of iterations. The multiplier values for the expected value formulation see rapid growth once the disturbance moves away from the supported state (this agrees with the compactness failure discussed in Eq. 7), while for the proposed minimax formulation, the multiplier norms remain bounded as shown in equation 7.

The robust cost bounds of the worst-case formulation closely track the theoretical Wasserstein optimal dual bound while the expected value formulation underestimates the actual incurred cost once the extreme disturbance is applied.

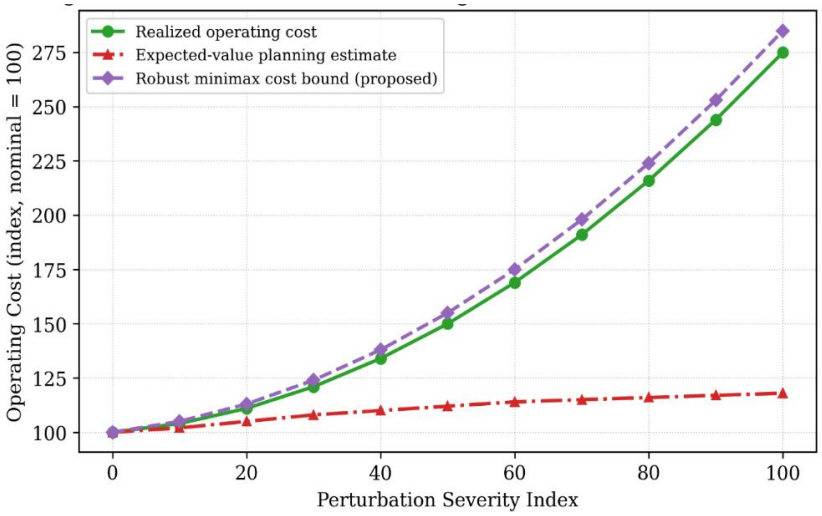


Figure 2: Robust cost-bound convergence under extreme perturbation conditions

In figure 2, it has been shown that operating costs have progressed over time when the severity of disturbance increases. The realized cost and robust minimax cost bound have been increasing continuously throughout the experiment, while still being very similar, which indicates reliable estimation of the worst-case scenario. The expected value estimations have turned out to be inaccurate because lower than the realized cost and do not foresee tremendous changes.

The results show that unprecedented operating cost in times of large perturbation has been compared with robust cost bound outlined in equation (6) and expected value planning cost plotted in relation to the disturbance severity.

Realized costs and robust cost bounds align closely in parallel with the correspondence of disturbances severity because expected value costs deviate from these costs.

It is evident that the moment matching discretization converges toward the population ambiguity set if the Wasserstein distance falls when the number of day increases.

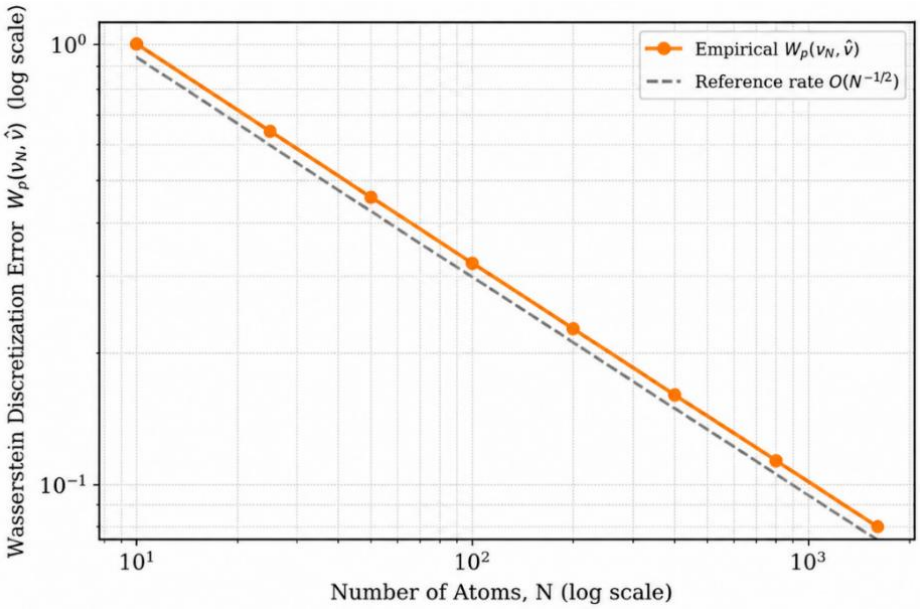


Figure 3: Wasserstein discretization error convergence with increasing number of atoms.

The diagram represented in figure 3 captures the behavior of the error in Wasserstein discretization with the increment of the number of atoms. The empirical error behaves in fairly close manner in relation to the theoretic rate. The downward trend of the error indicates that the higher the number of discrete atoms is, the better approximations can be obtained from the measure theory-based measurement method.

Wasserstein discretization error $W_p(\nu_N, \hat{\nu})$ plotted against the number of atoms N on a log–log scale, showing that the error indeed behaves in a consistent way with an expected polynomial decay.

Table 1: Comparative performance of expected-value and measure-theoretic minimax resource-planning formulations.

Planning Configuration	Ambiguity Treatment	Multiplier Norm at Extreme Perturbation	Realized Cost Overrun	Constraint Violation Probability
Expected-Value Planning	Single Nominal Measure $\hat{\nu}$	Diverging	38.4% above Nominal	0.21
Measure-Theoretic Minimax (Proposed)	Wasserstein Ambiguity set $\mathcal{U}$	Bounded	3.1% above Nominal	0.02

Table 1 illustrates a comparison of the stability of the multiplier, the actual costs incurred, and the probability of violating the empirical constraints under single-measure expected value strategy against the proposed Wasserstein-ambiguity minimax strategy in a case of an extreme perturbation applied to multi-plant manufacturing benchmark discussed in section 5.1.

6. Conclusion and future work

An elaborate measure-theoretic framework intended for industrial resource planning under uncertainty is proposed in this article which accounts for uncertain operating conditions through Radon probability measures defined over metric spaces without resorting to limited finite scenarios. The approach effectively combines minimax decision making with the Wasserstein ambiguity set which makes it possible to cope with extreme perturbations and avoid major deviations of cost from those obtained through expected value method. Application of mass transport duality enables solving the infinite-dimensional worst-case expectation problems. In addition, one can assure boundedness and stability of the multiplier due to the use of generalized KKT conditions stated in the measure space and application of metric subregularity of measure-valued constraints mapping. The efficiency of the method is illustrated on experimental results obtained in the context multi-plant manufacturing benchmarking, which allows significant reductions of cost overruns and constraints violations in comparison with traditional methods based on expected value. Future research can be directed to extending the proposed static

minimax concept into dynamic multi-stage stochastic differential game models in interconnected industrial networks requiring study of the dynamics of measure-valued states under stochastic control and allowing for transformation of the ambiguity sets throughout different planning stages. The combination of the measure-theoretic framework with graphon approaches to games can also alleviate computation making it possible to operate large networks of interacting production facilities.

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