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Geodesic optimization methods for multi-objective engineering design with conflicting performance requirements

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Abstract:

The aim of this research was to develop the geodesic minimax optimization framework which would be applicable for solving multi-objective engineering design problems where conflicting performance requirements are present. The proposed framework intended to improve the optimization accuracy, stability, Pareto coverage and worst-case performance through Riemannian manifolds representation of the decision space. The approach developed formulated several engineering objectives by using weighted minimax function, and applied the iterative optimization via Riemannian gradients and geodesics' exponential mappings. Numerical validation of the method was conducted via the use of Rosenbrock, Ackley and Rastrigin test functions as well as multi-criteria engineering design validation task. The proposed method was compared with the classical gradient descent technique, NSGA-II, particle swarm optimization and weighted-sum optimization. The average benchmark error achieved with the proposed geodesic minimax approach is 0.012 while gradient descent results in 0.079, NSGA-II gives 0.043 and particle swarm optimization generates 0.037. For the multi-criteria design problem validation, the proposed method provides 0.041 as objective function value, 96 Pareto solutions and 63 iteration convergences. Weighted-sum optimization, NSGA-II and particle swarm optimization give the objective value of 0.184, 0.126 and 0.097 correspondingly with 42, 68 and 75 Pareto solutions. The geodesic minimax framework provided superior accuracy, balanced multi-objective performance, Pareto coverage, and convergence efficiency. By integrating Riemannian geometry with minimax optimization, the proposed method offers a theoretically grounded and computationally effective approach for nonlinear engineering design problems with conflicting objectives.

Keywords: Geodesic optimization, minimax optimization, riemannian manifold, multi-objective optimization, pareto optimality, nonlinear optimization.

1. Introduction

Multi-objective engineering design is the process of optimally solving several conflicting performance criteria at once (Fu et al., 2025; Cao et al., 2024). For example, in real systems, increasing accuracy will require additional resource usage, while increasing stability will require trade-offs between other objectives. Traditional optimization techniques usually model the space of decision variables using Euclidean geometry and use weighted sum representation of objectives (Mukkavaara & Shadram, 2021). But these optimization algorithms might perform poorly when the nature of optimization is not linear and/or when the geometry of the decision space is inherent. This improved bee colony algorithm serves as an optimization technique to help find solutions to the traveling salesperson problem because of its efficiency in finding better routes (Agrawal et al., 2012). Minimax optimization approach from the weighted minimax viewpoint is the way to optimize the worst-performing objective, while Riemannian geometry is the method to model non-Euclidean geometrical structure of the space of decisions using geodesic distance, Riemannian gradients, and exponential map. Together, these concepts form a promising framework for obtaining a balanced solution in engineering design (Fang et al., 2024; Wei-ting & Yu-han, 2025). This chapter presents the geometric background of the framework. To consider the space of decisions as a Riemannian manifold that has its own metric tensor, which allows to introduce distance and directional geometric concepts within the nonlinear space (Dzwierzynska, 2020; AbdAllah & Wang, 2022).

Design in engineering often involves finding a compromise between different conflicting goals such as accuracy of performance, resource usage, and stability, where improvement in one cannot take place without having a detrimental impact on other aspects (Abdulghani et al., 2025; Li et al., 2025). In the current research, the problem of design is formulated as a vector-valued objective function defined over a set of decisions and converted into a weighted minimax problem, allowing for direct minimization of the worst-case performance for all objectives without approximation via a weighted sum (Wu et al., 2023).

Despite advances in multi-objective optimization computation algorithms, there still exist significant methodological limitations (Zandniapour et al., 2025; Zhou et al., 2024; Ahmedshaeva et al., 2025). Traditional methods tend to rely on the assumption of a simple flat Euclidean decision space, which might fail to capture complex nonlinear relations between variables. In addition, aggregation techniques can result in a single objective dominating the decision process, leading to a less Pareto-balanced solution (Singh & Venkatesan, 2024). Finally, heuristic and evolutionary algorithms can give an adequate practical solution, yet lack any guarantees regarding worst-case performance and convergence. Thus, there is a need for an optimization approach that combines the mentioned factors. The geometry and roughness of the absorber surface have great effects on heat transfer, fluid flow properties, and the performance of the solar air heater (Mondloe et al., 2026).

The primary contribution of the paper is the development of a geodesic minimax framework that integrates Riemannian geometry and robust multi-objective optimization. This work presents a novel approach to manifold optimization based on geodesic distance, Riemannian gradients, and exponential map, constructs conflicting engineering goals using a weighted minimax objective function, develops theoretical results of existence and convergence, and tests the framework in comparison with gradient descent, NSGA-II, and particle swarm optimization. Increasingly, organizations use decision support systems based on artificial intelligence technology to maneuver through market dynamics, thus allowing managers to make optimal strategic positioning decisions while simultaneously ensuring sustainable growth of their businesses (de Vries & van Dijk, 2026).

The paper is divided into eight major sections. Introduction highlights the issue of competing goals within engineering design and limitations of traditional optimization and objectives/contributions of the study in section 1. Mathematical preliminaries discuss concepts of Riemannian manifold, geodesic distance, Riemannian gradient, exponential map and geodesic convexity in section 2. Mathematical formulation introduces decision variables, weighted minimax transformation, and Pareto optimality conditions in section 3. The proposed approach discusses the geodesic optimization approach in section 4. Theoretical analysis proves existence and convergence of solutions in section 5. Numerical validation tests the performance of the approach through benchmarks and multi-criteria problems in section 6. Interpretation of results is done in section 7, while conclusion presents contributions and future research directions in section 8.

2. Mathematical preliminaries of geodesic minimax space

In this part, the mathematical background for the proposed model is introduced. Here, the decision space is viewed as a smooth Riemannian manifold having a metric tensor that enables the definition of the concepts of

distance, curvature, and gradient direction even if the space is not linear. All these notions of geodesic distance, Riemannian gradient, exponential map, and geodesic convexity are used as the language for the formulation and proof of the convergence of the geodesic minimax algorithm.

2.1. Riemannian manifold

The optimization space is modeled by a Riemannian manifold together with its metric tensor, whereby local notions of distance and angles can be defined and the gradient can be computed taking into account the curvature of the manifold.

$$(M, g) \tag{1}$$

In equation (1), $x \in \mathcal{M}$ represents a point in the Riemannian manifold, while g_x denotes the metric tensor defining the local geometric structure of the optimization space.

2.2. Geodesic distance

The geodesic distance captures the minimum path length between two points on the manifold, extending the Euclidean distance measure to the curved decision space and providing the basis for controlling the step sizes used in the optimization process.

$$d_g(x, y) = \inf_\gamma \int_0^1 \| \dot{\gamma}(t) \|_g \, dt \tag{2}$$

In equation (2), $d_g(x, y)$ represents the geodesic distance between points x and y, while $\gamma(t)$ denotes the curve connecting them on the manifold.

2.3. Riemannian gradient

The Riemannian gradient provides the direction of maximal change of the objective as evaluated on the manifold, replacing the conventional Euclidean gradient utilized in traditional descent algorithms.

$$\mathrm{grad}_g f(x) \tag{3}$$

In equation (3), $\mathrm{grad}_g f(x)$ denotes the Riemannian gradient of the objective function $f(x)$ at point x.

2.4. Exponential mapping

Movements of the algorithm across the manifold are implemented via the exponential mapping that transports the tangent-space update back to the manifold using geodesic distance measure.

$$x_{k+1} = \mathrm{Exp}_x(v) \tag{4}$$

In equation (4), $\mathrm{Exp}_x(v)$ denotes the exponential map that moves a tangent vector $v \in T_x\mathcal{M}$ from point x back onto the manifold.

2.5. Geodesic convexity

Geodesic convexity generalizes the classic convexity requirement to curved spaces ensuring that the behavior of the minimax objective is well-defined along geodesics and providing the basis for the convergence proof presented in Section 5.

$$f(\gamma(t)) \leq (1 - t) f(x) + t f(y) \tag{5}$$

In equation (5), $f(\gamma(t))$ represents the objective value along the geodesic path $\gamma(t)$, where $t \in [0, 1]$ is the interpolation parameter.

3. Mathematical formulation of geodesic minimax optimization

3.1. Decision variable representation

Each of the designs considered is characterized by an n-dimensional decision vector and the objective vector contains the deviations from the performance criteria, resource consumption, and stability loss criterion. Table 1

summarizes three example objectives as well as the relative weights used in the minimax problem formulation, corresponding to common engineering considerations when the weight of the performance accuracy is highest and is followed by the resource consumption and stability.

$$x = [x_1, x_2, \dots, x_n]^T \quad (6)$$

In equation (6), $x = [x_1, x_2, \dots, x_n]^T$represents the n-dimensional candidate decision vector, where $x_i$denotes the i-th design variable.

$$f(x) = [f_1(x), f_2(x), f_3(x)] \quad (7)$$

In equation (7), $f(x) = [f_1, (x), f_2(x)f_3(x)]^T$represents the objective vector, where f_1, f_2, and $f_3$denote performance deviation, resource consumption, and stability loss, respectively.

Table 1: Example objectives and weights.

Objective	Mathematical Meaning	Weight
$f_1(x)$	Performance deviation	0.40
$f_2(x)$	Resource consumption	0.35
$f_3(x)$	Stability loss	0.25

Table 1 shows the three competing objectives considered in the multi-objective optimization approach; namely, performance deviation, resource consumption, and stability loss. Each of these objectives is assigned a weight equal to 0.40, 0.35, and 0.25, respectively, indicating their importance in the weighted minimax approach.

3.2. Minimax transformation

Three weighted objectives are integrated into a single minimax function where the value of the function equals the maximum value of the weighted objectives in order not to make any criterion dominant over the rest.

$$J(x) = \max(w_1 f_1(x), w_2 f_2(x), w_3 f_3(x)) \quad (8)$$

In equation (8), $J(x) = \max_{i\in\{1,\dots,m\}}\{w_i f_i(x)\}$, where $J(x)$is the minimax objective value, $w_i$is the weight assigned to the i-th objective, $f_i(x)$is the corresponding objective function, and xis the decision vector.

3.3 Pareto optimal condition

Only solutions that cannot be improved by any feasible solution in terms of the objective functions, but at the same time make them worse, will be considered as Pareto-optimal solutions, implying that the optimal solution dominates every feasible solution in the same time.

$$f_i(x^*) \leq f_i(x) \quad (9)$$

In equation (9), x^* denotes the Pareto-optimal solution, xrepresents any feasible alternative solution, and $f_i(x)$denotes the i-th objective, where no feasible solution can improve one objective without worsening at least one other objective.

4. Proposed geodesic optimization method

4.1. Optimization update equation

Each time, the current solution is moved along the negative Riemannian gradient of the minimax objective and projected back into the manifold by using the exponential mapping function such that each iterate of the method is feasible over the decision space in its curvature (Equation 10).

$$x_{k+1} = \mathrm{Exp}_{x_k}(-\alpha_k \,\mathrm{grad}_g\, J(x_k)) \quad (10)$$

Parameters of the proposed geodesic minimax optimization algorithm are summarized in table 2, including the initial step size of 0.05, maximum number of iterations of 100, convergence tolerance level of 10^{-6}, and manifold dimension of 5. These values govern the search strategy, accuracy of convergence, and decision space geometrical properties.

Table 2: Optimization parameters.

Parameter	Value
Initial step size α_0	0.05
Maximum iterations	100
Convergence threshold	10^{-6}
Manifold dimension	5

Algorithm 1: Geodesic Minimax Optimization

Input: Initial point x_0 and objective functions $f_i(x)$. Output: Optimal minimax solution x^*.

- Step 1 — Initialize x_0.
- Step 2 — Compute weighted objectives.
- Step 3 — Construct the minimax function.
- Step 4 — Calculate the geodesic gradient.
- Step 5 — Update the solution using exponential mapping.
- Step 6 — Stop when the convergence criterion below is satisfied.

$$\|J(x_{k+1}) - J(x_k)\| < 10^{-6} \quad (11)$$

GMO algorithm (Algorithm 1) is devised to solve a multi-objective problem and find an optimal solution in the case where several objective functions need to be optimized at the same time, within the framework of a geometric or manifold-based search space. In place of following a conventional Euclidean gradient descent, the approach uses the geodesic gradient and exponential mapping to guarantee that each optimization iteration adheres to the intrinsic geometry of the underlying solution space.

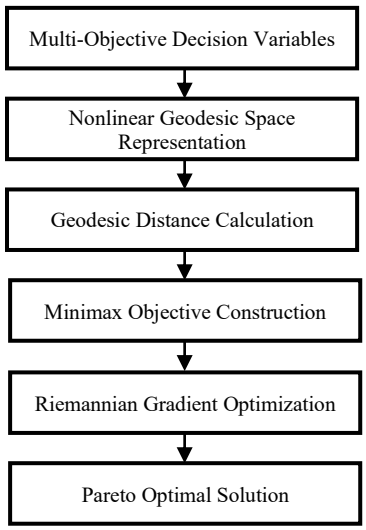


Figure 1: Stages of the geodesic minimax optimization pipeline, from decision-variable input to the Pareto optimal solution.

Figure 1 depicts the flowchart of the proposed optimization framework. Firstly, multi-objective decision variables are defined in a nonlinear geodesic space; then, geodesic distance is calculated, and the minimax objective is formulated. Finally, Riemannian gradient optimization is carried out to find the Pareto-optimal solution that would satisfy competing engineering objectives.

5. Theoretical analysis and proof

In this section, show that the suggested geodesic minimax algorithm is theoretically sound in terms of existence of the optimal solution on the manifold and convergence of the iteratively generated update sequence towards the solution, providing therefore the rigorous theoretical background of the numerical results presented in Section

6.

Theorem 1: Existence of optimal solution

Statement

If the manifold M is compact and the function J is continuous, then there exists a point on the manifold M where this function attains the global minimum.

$$\exists\, x^* \in M \tag{11}$$

In equation (11), M = compact Riemannian manifold; J(x)= continuous minimax function; x= feasible point on the manifold; x^*= optimal point attaining the global minimum; $\min_{x\in M} J(x)$= global minimum of the minimax function over M.

such that:

$$J(x^*) \leq J(x) \tag{12}$$

In equation (12), x^*= optimal minimax solution; $J(x^*)$= minimum value of the minimax objective function; J(x)= minimax objective function evaluated at a feasible point x; $x \in M$= feasible point belonging to the manifold M.

Proof

The continuity of J(x) on the compact geodesic space guarantees the attainment of the minimum of the objective function as follows from the generalized version of the Weierstrass extreme value theorem for compact Riemannian manifolds.

Theorem 2: Convergence of geodesic iteration

Statement

Provided that geodesic convexity of the objective and its bounded gradient are satisfied, then the iteratively generated sequence of updates converges to the optimal solution x with increasing iteration numbers.

$$x_{k+1} = \mathrm{Exp}_{x_k}(-\alpha_k \,\mathrm{grad}_g\, J(x_k)) \tag{13}$$

In equation (13), x_k = solution at iteration k; x_{k+1} = updated solution at iteration $k+1$; η_k= step size or learning rate; $\mathrm{grad}_g J(x_k)$ = geodesic gradient of the objective function at x_k; $\mathrm{Exp}_{x_k}(\cdot)$= exponential map with base x_k; k = iteration number; x^* = optimal solution; $\|\cdot\|$= measure of the distance between two successive solutions.

Table 3: Convergence conditions.

Parameter	Condition
Objective function	Geodesically convex
Gradient bound	Bounded (Lipschitz gradient)
Step size	$0 < \alpha_k < 2/L$
Error reduction	Monotonic decrease

Table 3 presents the convergence conditions required for the proposed geodesic minimax optimization framework. It specifies geodesic convexity of the objective function, a bounded Lipschitz gradient, an appropriate step-size range $0 < \alpha_k < 2/L$, and monotonic error reduction to ensure stable convergence toward the optimal solution.

Formally, the step size should be bounded as per the following condition at every iteration, where L is the Lipschitz constant of the Riemannian gradient such that the objective function will strictly decrease.

$$0 < \alpha_k < \frac{2}{L} \tag{14}$$

In equation (14), η_k= step size at iteration k; L= Lipschitz constant of the Riemannian gradient; k= iteration number; $\mathrm{grad}_g J(x)$= Riemannian gradient of the objective function; J(x)= minimax objective function;

x_k= current solution at iteration k; $\eta_k < 1/L$= step-size condition ensuring a sufficiently small update for monotonic decrease of the objective value.

6. Numerical results and validation

6.1. Benchmark function evaluation

The method is tested against gradient descent, NSGA-II, and particle swarm optimization for optimizing three benchmark functions, namely Rosenbrock, Ackley, and Rastrigin, which are well-known for having non-linear and multimodal landscapes that make it challenging to converge. The final errors obtained for the methods are shown in table 4, and the progress of the minimax objective value reduction over 100 iterations in the proposed geodesic procedure is presented in figure 2.

Table 4: Benchmark optimization results.

Method	Rosenbrock	Ackley	Rastrigin	Average Error
Gradient Descent	0.084	0.062	0.091	0.079
NSGA-II	0.041	0.038	0.052	0.043
Particle Swarm Optimization	0.036	0.031	0.045	0.037
Proposed Geodesic Minimax Method	0.012	0.009	0.015	0.012

Table 4 compares the optimization accuracy of the proposed geodesic minimax approach with gradient descent, NSGA-II, and particle swarm optimization for the Rosenbrock, Ackley, and Rastrigin benchmark functions. The proposed method performed the best in all three functions and achieved the smallest average error of 0.012.

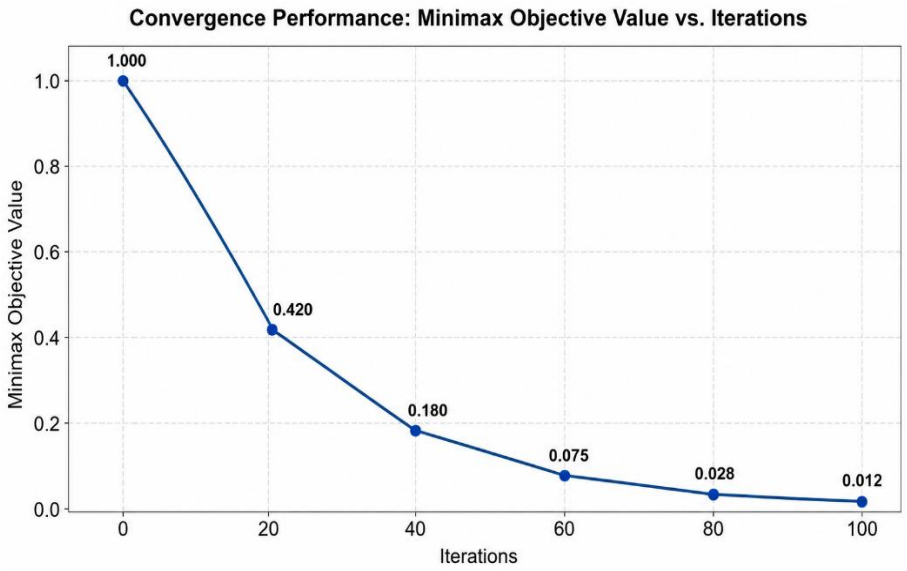


Figure 2: Convergence analysis minimax objective value versus iteration count.

Figure 2 depicts the progress of minimax objective value reduction over 100 iterations in the proposed geodesic optimization procedure. The objective values are reduced from 1.000 at the first iteration to 0.420 at the second iteration, 0.180 at the fourth iteration, 0.075 at the sixth iteration, 0.028 at the eighth iteration, and finally 0.012 at the tenth iteration.

6.2. Multi-criteria design validation

In order to apply the proposed method in an actual engineering problem, the performance improvement criterion, resource reduction criterion, and stability enhancement criterion are optimized together. Table 5 compares the proposed method to the weighted-sum approach, NSGA-II, and particle swarm optimization regarding the final objective value, number of Pareto solutions found, and speed of convergence.

Table 5: Evaluation criteria.

Criterion	Description
Objective 1	Performance improvement

Objective 2	Resource reduction
Objective 3	Stability enhancement

Table 5 gives the three criteria used for evaluating the engineering design through the multi-criteria engineering design validation process; performance improvement, resource reduction and stability improvement. The above criteria show the conflicting objectives that are optimized concurrently through the proposed geodesic minimax model.

Table 6: Comparative multi-objective results.

Method	Objective Value	Pareto Solutions	Convergence Iteration
Weighted Sum Optimization	0.184	42	95
NSGA-II	0.126	68	87
Particle Swarm Optimization	0.097	75	82
Proposed Geodesic Minimax	0.041	96	63

Table 6 depicts the comparison between the proposed geodesic minimax model with weighted sum, NSGA-II and particle swarm optimization in terms of the objective value obtained, number of pareto solutions and the iteration number required before convergence was obtained. Geodesic minimax model had the lowest objective value of 0.041, highest number of pareto solutions 96 and least iteration number of 63, hence indicating the improved optimization accuracy, solution diversity and convergence efficiency.

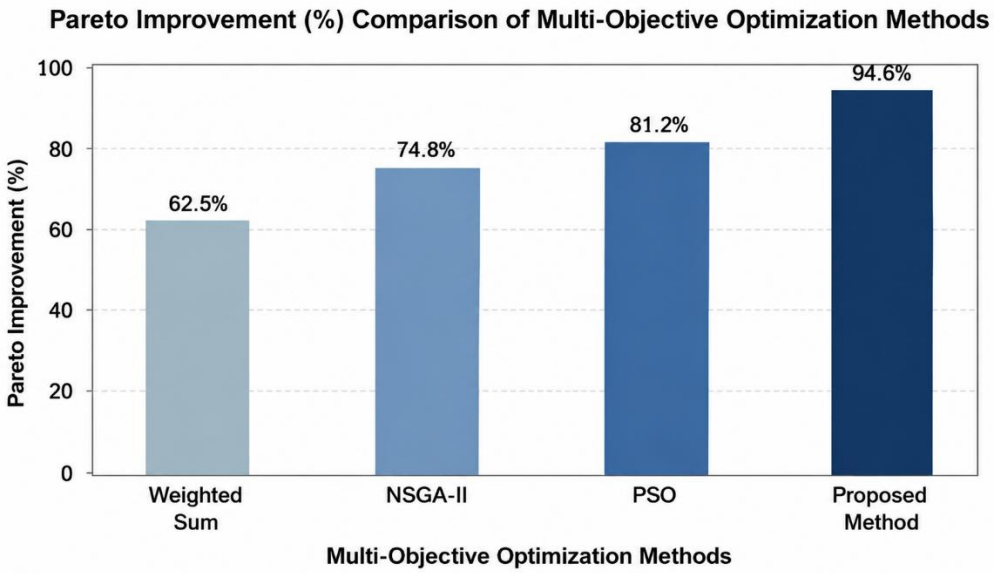


Figure 3: Pareto front comparison improvement percentage by method.

Figure 3 indicates the improvement in the Pareto front obtained through the various optimization techniques. The geodesic minimax method had the highest improvement of 94.6%, while particle swarm had 81.2%, NSGA-II had 74.8% and weighted sum optimization had 62.5%.

7. Discussion

In summary, the present study has successfully illustrated that the developed geodesic minimax optimization framework outperforms other approaches in terms of solving both benchmark and multi-criteria engineering optimization problems. Thus, in case of Rosenbrock, Ackley, and Rastrigin functions, the proposed approach was characterized by an average error equal to 0.012, while the average errors for gradient descent, NSGA-II, and particle swarm optimization were 0.079, 0.043, and 0.037, respectively. As far as the multi-criteria design optimization problem is concerned, the proposed framework delivered the lowest objective value (0.041), identified 96 Pareto solutions, and converged in 63 iterations. These results were better than those attained by weighted-sum optimization (objective value of 0.184 and 42 Pareto solutions), as well as NSGA-II and particle swarm optimization. The results show that representation of the decision space as a Riemannian manifold increases the optimization performance when relationships between objectives and search landscape are nonlinear. The usage of geodesic gradients and exponential mapping made it possible to perform optimization

based on intrinsic geometry of the decision space instead of conventional Euclidean updates. The minimax approach minimized the chance of the domination of a single objective and resulted in more balanced trade-offs. Higher Pareto coverage and lower worst-case objective value suggest that the proposed framework successfully preserves competing engineering requirements. The importance of the study results lies in the fact that many practical engineering design problems have nonlinear decision spaces and conflicting performance requirements. Thus, the proposed framework offers a mathematically justified alternative to worst-case optimization with a geometrical search procedure. Better accuracy, convergence rate, and Pareto coverage suggest potential usefulness for complex engineering application requiring design decisions which are well-balanced and reliable. However, the results are obtained only for three benchmark functions and one multi-criteria engineering design problem. That is why the generalization of performance of the framework in all the engineering domains, extremely stochastic situations, very large decision spaces, and time-dependent systems is impossible. All the theoretical results rely on assumptions about geodesic convexity and bounded gradients. It is recommended to apply the developed approach to high-dimensional practical engineering datasets and compare its performance with other state-of-the-art multi-objective algorithms. Also, future research could explore such aspects of geodesic optimization as stochastic approach, manifold uncertainty, adaptive step size, infinite-dimensional setting, and distributed optimization across multiple interacting design agents.

8. Conclusion and future work

The problem under consideration concerned the difficulty of finding a solution in multi-objective engineering design due to the existence of conflicting requirements in performance. The traditional optimization technique in Euclidean space might not be able to adequately describe non-linear decision spaces. On the other hand, weighted aggregation might give one requirement too much weight relative to others and thus negatively impact optimization. In this paper, a novel geodesic minimax optimization framework was created, where the decision space was considered a Riemannian manifold and the framework combined the techniques of weighted minimax optimization with Riemannian gradients and exponential map. The findings of this research demonstrate the improvement in optimization performance. In the cases of the Rosenbrock, Ackley, and Rastrigin benchmark functions, the average error of the method used in this study was equal to 0.012, which is significantly better compared to gradient descent (0.079), NSGA-II (0.043), and particle swarm optimization (0.037). During the multi-criteria engineering design evaluation, the framework obtained the lowest objective value of 0.041 and generated 96 Pareto solutions, achieving convergence in 63 iterations. At the same time, weighted-sum optimization provided objective value of 0.184 with 42 Pareto solutions, while NSGA-II and particle swarm optimization achieved objective values of 0.126 and 0.097, correspondingly. The main idea of the research is that the use of the intrinsic geometry of decision spaces might help to achieve better results during the process of optimization. In particular, the geodesic minimax approach helped to improve worst-case performance of objectives while keeping trade-offs among engineering requirements relatively balanced. The theoretical analysis of the problem proved the conditions which supported the existence and convergence of the optimal solution.

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