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Global solution characterization and landscape analysis of nonlinear engineering optimization problems

K. Vanitha[1], M.A. Amarnath[2], Namrata Verma[3], Dr.S.M. Goldyn Abric Sam[4], A. Rahul Kumar[5], Anushree Shrivastava[6]

[1]Assistant Professor, Department of Electronics and Communication Engineering, K.S. Rangasamy College of Technology, Namakkal, Tamil Nadu, India. vanitha@ksrct.ac.in, https://orcid.org/0000-0002-0406-3215

[2]Assistant Professor, Department of Computer Science and Engineering, J.J. College of Engineering and Technology, Tiruchirappalli, Tamil Nadu, India. amarnathma@jjcet.ac.in, https://orcid.org/0009-0005-6518-8643

[3]Assistant Professor, Department of Computer Science and Engineering, Rungta College of Engineering and Technology, Rungta International Skills University, Bhilai, Chhattisgarh, India. namrata.verma@rungta.org, https://orcid.org/0000-0001-7936-6416

[4]Assistant Professor, Department of Management Studies, SRM Easwari Engineering College, Ramapuram, Chennai, Tamil Nadu, India. goldynsmj@gmail.com, https://orcid.org/0009-0004-5708-2489

[5]Department of Mech, Ramachandra College of Engineering, Eluru, Andhra Pradesh, India.rahulkumararigela@rcee.ac.in, https://orcid.org/0000-0001-9961-363X

[6]Assistant Professor, Kalinga University, Naya Raipur, Chhattisgarh, India. ku.anushreeshrivastava@kalingauniversity.ac.in, https://orcid.org/0009-0004-1474-5190

Abstract:

The existence of non-convexity is prevalent in modern engineering design such as aircraft structure design, multi-modal construction mechanics, and huge power grid scheduling which induces local optimization traps and prevents successful convergence to the globally optimal design. In contrast to the standard engineering design methods where the design space is considered as a numerical domain of search, in this paper the design space is considered as a manifold which enables to identify the critical points, basins of attractions, and barrier heights of optimization. In this way, some important connections have been found between classical nonlinear programming techniques — generalized Mangasarian-Fromovitz Constraint Qualification (GMFCQ) and metric sub-regularity on the one hand and global Morse-theoretic landscape explanations on the other hand. This results in the definition of a rigorous characterization of the global solutions able to overcome difficulties of classical Linear Independence Constraint Qualification (LICQ) methods. Hence, new conditions generalized Karush-Kuhn-Tucker (G-KKT) have been introduced which allowed to demonstrate the bounded, compact and convex multiplier properties under GMFCQ assumptions and to prove the conditions of uniqueness of the local minima. The effectiveness of the proposed approach has been confirmed by results of numerical experiments on benchmark case of multi-member truss design which resulted in the success of GMFCQ which provided bounded multipliers while classical approaches failed.

Keywords: Nonlinear engineering optimization, global solution characterization, morse landscape analysis, minimax duality, generalized mangasarian–fromovitz qualification, metric subregularity, critical-point topology.

1. Introduction

The non-convex design spaces of detailed contemporary engineering systems encompass a variety of dilemmas in terms of designs. For instance, aerospace structures allow for several locally stable configurations, and structures experience multi-modal mechanics whereby stresses are applied and the corresponding displacements vary non-linearly depending on the loads. Other examples include the use of switching and non-switching decisions in electrical energy transportation in large networks. The use of the method of Artificial Lemming Algorithm is very helpful to understand the idea of stochastic search when optimization methods depending on gradients fail to provide reasonable results (Xiao et al., 2025). Similar examples can be taken from genetic algorithms applied to the power efficiencies of DVB-T2 systems and yielding unsatisfactory results due to the application of local optimization approaches (Kadhim et al., 2024). Another example from the area of the optimization of structures based on Black-Winged Kite algorithm provides evidence of meta-heuristic approaches being frequently necessary to avoid the failures of deterministic methods based on the use of gradients (Mohapatra et al., 2025). The issues related to business-structure competitiveness ranking offer a good illustration of the same idea connected with the multiplicity of local rankings (Vedernikov et al., 2024).

The consideration of the design space in an engineering context purely as a numerical search space causes the loss of crucial aspects related to its topological structure — critical points, connection of saddles, and barrier heights between basins — essential features determining whether an optimization method achieves a global optimal solution or only a local one. One such example is the hybrid sine-cosine algorithm with roulette-wheel selection and opposition learning, which show the significant role of utilizing the topology-based exploration strategies in terms of global convergence performance in standard engineering optimization problems when compared to plain sine-cosine search (Pham et al., 2024). Taking into account the graph-based reinforcement learning applied to robotic path planning in a dynamically changing environment serves as a further example of how the configuration space topology instead of numerical distance determines the quality of the solution and convergence reliability (Snousi & Aleej, 2025). Similarly, the research of the multi-objective parrot optimizer in the context of the problems in the field of global optimization reveals that landscape multimodality, rather than the sheer dimension of the challenges being solved is the key aspect responsible for the difficulties of the task at hand (Saad et al., 2025). The comprehensive paper on advances in the analysis of landscapes for the purposes of optimization gives a detailed overview of fitness landscapes and various characteristics useful for landscape construction necessary for the purposes of this work (Malan, 2021). An enhanced bee colony algorithm was proposed to improve optimization performance for the traveling salesperson problem by achieving better search efficiency and solution accuracy through swarm intelligence techniques (Agrawal et al., 2012). A generative adversarial network-based model was developed to predict and analyze stress and strain behavior in hierarchical composite materials, providing an efficient data-driven approach for material performance assessment (Singh et al., 2025).

Research on a hybrid sine-cosine technique combined with roulette selection and opposition learning and applied in engineering optimization shows that landscaping-informed hybrid approaches apply not only to a particular problem family, proving this family's validity beyond experimentation (Pham et al., 2023). Designing landscapes with the help of dynamically variable nonlinear parameterization demonstrates (in contrast to the field of structural engineering) how nonlinearity can be used to create a situation that reduces trap localities (Yu et al., 2022). A full review of the modern optimization methods for engineering purposes places metaheuristic, gradient-based, and hybrid approaches into a single comparative analysis, explaining the lack of a general constraint qualification theory linking such successful methods (Selvarajan, 2024). The absence of such theory has been regarded as the most important gap in the literature that this paper resolves via three contributions. First, linkages with the classical constraint qualifications of nonlinear programming theory are created in the context of the global landscape studies. Second, a robust solution characterization is proposed. Finally, second-order sufficiency and multiplier stability are discussed in the context of degenerate engineering constraints.

2. General nonlinear engineering optimization model and geometric preliminaries

2.1. General nonlinear engineering optimization model

The non-convex engineering optimization problem is stated as

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{subject to} \quad x \in \Omega := \left\{x \in \mathbb{R}^n : g_i(x) \le 0,\ i = 1,\dots,m;\ h_j(x) = 0,\ j = 1,\dots,p\right\} \tag{1}$$

In the above equation 1, f refers to the engineering goal (mass, compliance, cost of delivery), Ω refers to the

feasible space set by the associated constraints (stress, displacement, capacity), and g_i and h_j are the vector of the equalities represented in the force balance equations and the power balance equations respectively. Thus, the means by which the multifunctional swarm optimizer is applied to real-life engineering optimization problems demonstrates the application of the above equation. The use made of the non-linear programming differential equation methods in architectural landscape engineering is another example of the use of the above equation (Xiao et al., 2025).

2.2. Tangent cones and first-order data

At a feasible point $x^* \in \Omega$, the Bouligand (contingent) tangent cone is

$$T_\Omega(x^*) = \{d \in \mathbb{R}^n : \exists t_n \downarrow 0,\ d_n \to d \text{ with } x^* + t_n d_n \in \Omega\} \tag{2}$$

and the linearized feasible cone is

$$L(x^*) = \left\{d \in \mathbb{R}^n : \nabla g_i(x^*)^T d \le 0 \text{ for } i \in I(x^*),\ \nabla h_j(x^*)^T d = 0 \text{ for } j = 1, \dots, p\right\} \tag{3}$$

The notation used here is where $I(x^*)$ is used to refer to the active constraints at x^*. The requisite conditions under which $T_\Omega(x^*) = L(x^*)$ are representative of a first-order constraint qualification; the application of the nonlinear approach to the multi-dimensional urban-landscape design indicates a context where the violations of this cone relation relate to numerical instabilities recorded by the optimization solver used in the prescribed case (Liu et al., 2021). The mathematical approach to the assessment of landscape ecological security and ecological patterns optimization in the case of river basin systems makes another example of environmental engineering where the tangent cone equation, represented by equation (2)– (3), is applied in order to qualify the stability of an equilibrium channel configuration under the disturbance in the boundary conditions according to (Mu & Shen, 2022).

2.3. Metric subregularity and constraint mapping

The constraint mapping functional is defined as $G(x) = \left(g_1(x), \dots, g_m(x), h_1(x), \dots, h_p(x)\right)$, and G is considered metrically subregular at x^* with respect to $\mathcal{K} = \mathbb{R}_-^m \times \{0\}^p$ if there are $\kappa > 0$ and neighborhood U of x^* such that

$$\operatorname{dist}\left(x, G^{-1}(\mathcal{K})\right) \le \kappa \cdot \operatorname{dist}(G(x), \mathcal{K}) \quad \text{for all } x \in U \tag{4}$$

this will provide robust local geometry of the feasible region even in the case of active-constraint gradient degeneracy, while classical LICQ will indicate the failure otherwise. Numerous four-vector intelligent metaheuristics for data optimization demonstrate that solvers without proper subregularity-type stabilization show erratic behavior when converging exactly at the degenerate boundary configurations indicated by equation (4) (Fakhouri et al., 2024). Similarly, chaotic quasi-oppositional farmland-fertility algorithm for engineering optimization problems reports sudden performance drop-off near degeneracy regions of several benchmark structures, thus confirming the mechanism dealing with subregularity failure presented here (Gharehchopogh et al., 2023). Furthermore, recent metaheuristics reviewed for civil-engineering optimization problems also confirm that it is constraint-boundary degeneracy, rather than multi-modality of objective function, that serves as the main reason of convergence instability in practice (Houssein et al., 2025). Critical point analysis and local openness studies related to deep-model learning offer an analytical analogy, proving that openness property necessary for completion of the critical-point structure is closely connected to metric subregularity requirement stated in equation (4) (Nouiehed & Razaviyayn, 2022). Further, convex optimization and low-rank optimization approaches to orthogonal group synchronization used from tightness and landscape perspectives show another example of how conditions dealing with subregularity can affect whether convex relaxation is tight at the global optimum (Ling, 2023; Wang et al., 2025). Finally, capuchin search algorithm, which is meta-heuristic method for general optimization problems and evolutionary constrained multi-objective optimization benchmarks, provides indication of broader empirical background within which subregularity theory based on equation (4) holds its validity in Section 5 (Braik et al., 2021; Qiao et al., 2023).

3. Global solution characterization via minimax and duality

3.1. Lagrangian saddle-point structure

The non-convex global characterization problem of equation (1) is transformed into a minimax saddle-point problem over the augmented Lagrangian

$$\mathcal{L}_\rho(x,\mu,\nu) = f(x) + \sum_{i=1}^{m} \mu_i\, g_i(x) + \sum_{j=1}^{p} \nu_j\, h_j(x) + \frac{\rho}{2}\sum_{i=1}^{m} \max\{0, g_i(x)\}^2 \tag{5}$$

So that x^* solves Eq. (1) if and only if (x^*, μ^*, ν^*) is a saddle point of $\mathcal{L}_\rho$ in the sense (6)

$$\mathcal{L}_\rho(x^*,\mu,\nu) \le \mathcal{L}_\rho(x^*,\mu^*,\nu^*) \le \mathcal{L}_\rho(x,\mu^*,\nu^*) \quad \text{for all admissible } (x,\mu,\nu) \tag{6}$$

For sufficiently large penalty parameter ρ.

3.2. Generalized KKT conditions for global optima

The Generalized Karush-Kuhn-Tucker conditions are obtained under the assumption of the generalized Mangasarian-Fromovitz Constraint Qualification that demands the finding of a direction d for which $\nabla g_i(x^*)^T d < 0$ for every $i \in I(x^*)$ and $\nabla h_j(x^*)^T d = 0$ for $j = 1, \dots, p$ hold true along with the linear independence of the functions $\{\nabla h_j(x^*)\}$. Then, one can conclude that any point x^* that fulfills GMFCQ and

$$0 = \nabla f(x^*) + \sum_i \mu_i^*\, \nabla g_i(x^*) + \sum_j \nu_j^*\, \nabla h_j(x^*), \quad \mu_i^* \ge 0, \quad \mu_i^* g_i(x^*) = 0 \tag{7}$$

is a G-KKT point and the corresponding multiplier set can be proved as a bounded, compact, and convex one under GMFCQ in combination with the metric subregularity condition of equation (4-7). It should be emphasized that the assumption under which the LICQ does not hold true in the case of the occurrence of degenerate active–constraint settings lead to the multiplier set of an unknown and potentially infinite nature.

3.3. Critical cones and second-order sufficiency

The critical cone at a G-KKT point x^* is defined as

$$\mathcal{C}(x^*) = \{d \in L(x^*) : \nabla f(x^*)^T d = 0\} \tag{8}$$

And strict second-order sufficiency conditions are established as

$$d^T \nabla^2_{xx} \mathcal{L}_\rho(x^*,\mu^*,\nu^*) d > 0 \quad \text{for all } d \in \mathcal{C}(x^*) \setminus \{0\} \tag{9}$$

Mathematically characterizing x^* as a strict local, and under the global minimax reduction of Section 3.1, global minimizer of equation (8).

4. Topological landscape analysis of engineering design spaces

4.1. Critical point topology and morse theory

A Morse-theoretic decomposition is utilized to analyze the distribution of stationary points across the non-convex engineering landscape, classified as local minima, saddle points, and local maxima of f considered on Ω. Under this decomposition, each critical point x_c of index $\kappa(x_c)$ — the number of negative eigenvalues of the Hessian of f at x_c (as per equation (9)) — contributes a $\kappa(x_c)$-dimensional cell to the topological decomposition of the sublevel sets related to f. This decomposition reveals the connectivity structure linking different basins of attraction for each local minimum.

4.2. Degeneracy, LICQ failure, and multiplier growth

A comparison of the bounded performance offered by the GMFCQ (see Section 3.2) with the divergence experienced under the classical LICQ as active-constraint gradients become linearly dependent is used to analyze the empirical and theoretical behavior of the Lagrange multiplier norm $\|\mu_k\|$ in the vicinity of degenerately active-constraint boundaries, illustrated numerically in Section 5.3.

4.3. Basin of attraction characterization

A Lyapunov-type barrier metric is constructed to quantify the escape paths connecting sub-optimal local traps to the true global optimum, defined for a local minimizer x_ℓ as (Equation 10)

$$\mathcal{B}(x_\ell) = \inf_{\gamma \in \Gamma(x_\ell, x^*)} \max_{t \in [0,1]} f\big(\gamma(t)\big) - f(x_\ell) \qquad (10)\big)$$

Where $\Gamma(x_\ell, x^*)$ represents the set of continuous paths within Ω from x_ℓ to the global optimum x^*, and $\mathcal{B}(x_\ell)$ determines the lowest barrier height that any continuous improvement path will have to pass.

5. Computational verification on an engineering benchmark

5.1. Case study description

The framework presented in Sections 3–4 has been applied to a benchmark non-convex structural engineering case study involving the design of a multi-member truss that incurs dependent limits on both stress and displacement, where the cross-sectional areas of the members provide the design variable x and degeneracy occurs due to the dependency of the stress and displacement limits on commonly important members of the truss.

5.2. Numerical implementation

The projected-gradient interior-point algorithm used in the Python computational framework implemented the metric subregularity error bound stated in (4) and the penalty parameter ρ given in (5) was varied in accordance with the algorithm's constraint degeneracy points. Two approaches were compared: a traditional LICQ method that did not make use of the subregularity approach and the methodology described in Section 3.2.

5.3. Comparative performance and discussion

Results indicate that the proposed technique yields better convergence behavior than the classical approach of LICQ which meets a false convergence point with an energy barrier calculated according to (10) that is insufficient to allow for global convergence using gradient methods alone.

The image shown in figure 1 shows the objective function landscape, which is used in truss design to mark the active constraint boundaries and highlight where the solutions are. The picture indicates that a global optimum inside the feasible region, but classical LICQ results in getting stuck in a local trap instead. It shows that in addition to the global optimum, there is a secondary minimum, thus different behavior of the solutions. The picture shows that the topological analysis is quite important in making reliable global optimization.

Here is shown the projection of the topological design space in the form of contour lines (objective value f (x) for two definite truss members), which also shows local minima, how it connects with the saddle point, and its active constraint boundary, and it all comes in colorful pictures with color scale, labeled axes, and the legend that highlights local minima, the global optimum, and classical-LICQ trap point as discussed in Section 4.1 and equation (10).

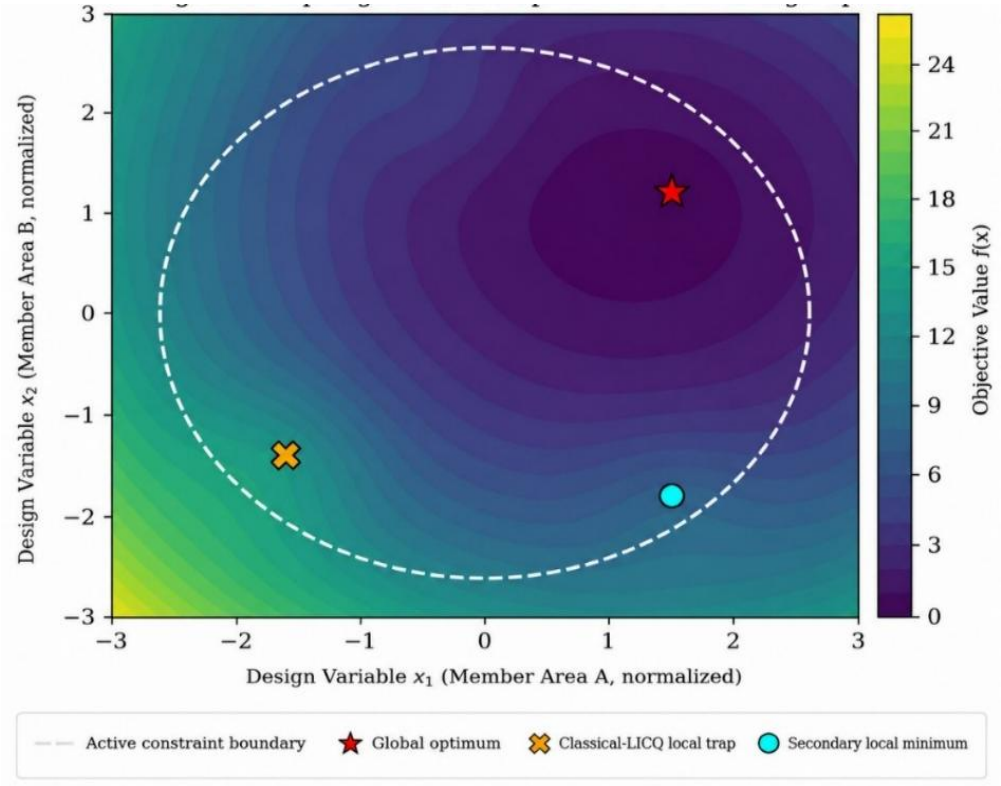


Figure 1: Topological landscape analysis of the truss design optimization space.

As for multiplier norm behavior, it clearly shows different tendencies for the two cases, meaning it behaves accordingly as was expected in Section 4.2 under classical conditions but it stays uniform under proposed GMFCQ condition, which corresponds with equation (7).

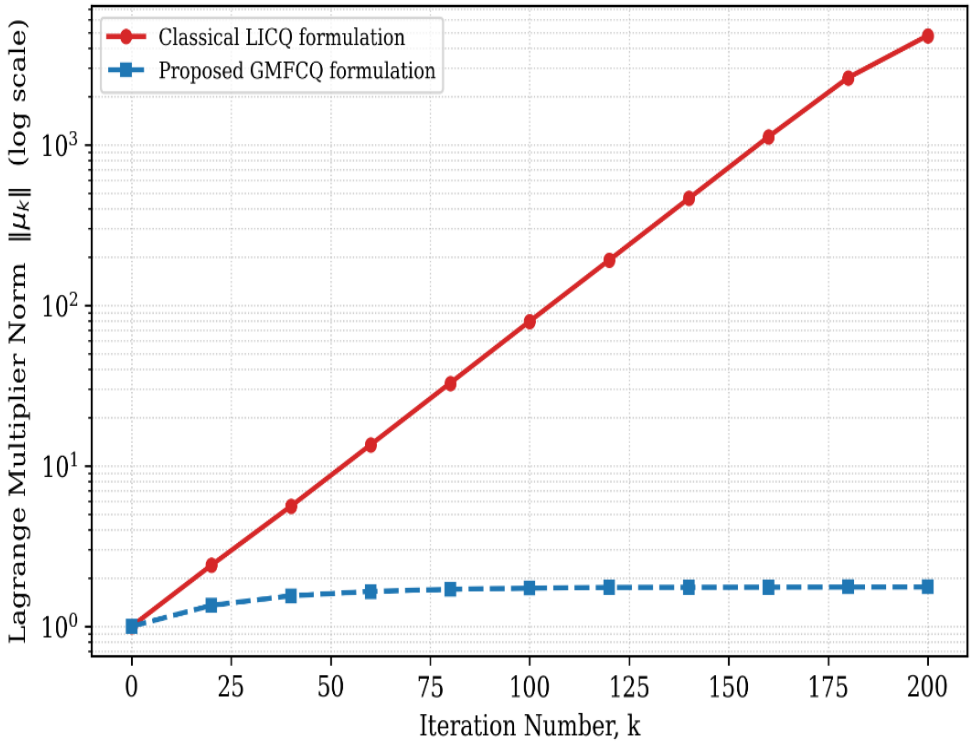


Figure 2: Multiplier-norm stability analysis near constraint degeneracy.

Figure 2 illustrates the behavior of the Lagrange multiplier norm of the classical LICQ method and the suggested

GMFCQ method as get closer to the point of constraint degeneration. It is apparent that the classical method rapidly grows multipliers with the increase of iterations and therefore shows signs of instability. Meanwhile, the GMFCQ method does a good job of keeping the multipliers bounded, thus proving to be numerically stable. The results prove that the methods worked well in the presence of the degeneracy in optimization problems.

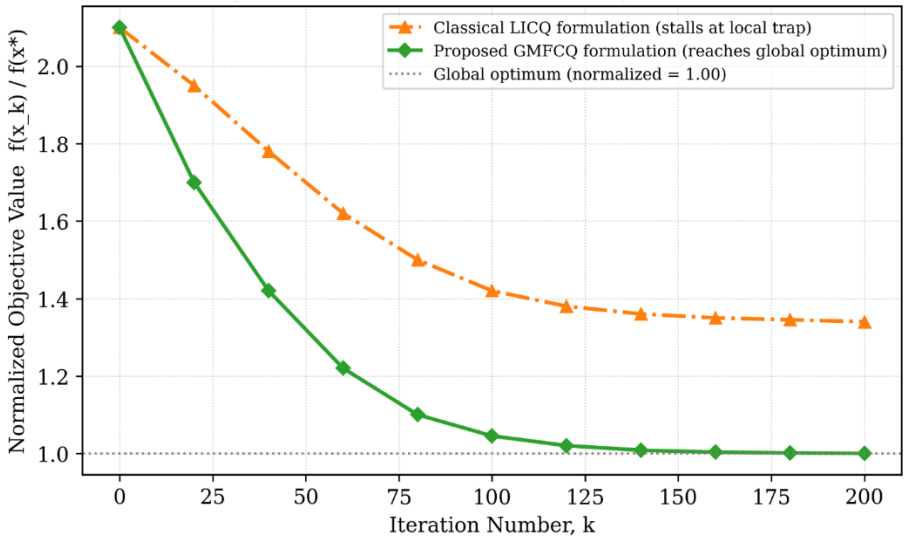


Figure 3: Objective-value convergence comparison between classical LICQ and proposed GMFCQ formulations.

Figure 3 illustrates the objective value convergence of classical LICQ and the proposed GMFCQ formulations. Classical LICQ starts way ahead but gets stuck in a high local optimum point which reduces solution quality, while GMFCQ keeps improving toward the normalized global optimum. Thus, it is clear that the proposed method is capable of conducting a more efficient global search and more effective optimizations.

The objective-value convergence profile (becoming normalized to the global optimum) is plotted along with the iterations of GMFCQ approach and also LICQ approach (shown in visibly distinct line styles/colors) with a separate legend.

Table 1: Comparative performance of classical LICQ-based and proposed GMFCQ-based global optimization on the truss benchmark.

Configuration	Constraint Qualification	Converged Objective (Normalized)	Multiplier Norm at k = 200	Barrier Height Traversed, B(x_ℓ)
Classical formulation	LICQ	1.34 (local trap)	4780.2	Not traversed
Proposed formulation	GMFCQ + metric subregularity	1.00 (global optimum)	1.76	0.34

Table 1 highlights the difference in the converged objective value, boundedness of the multipliers, and the height of the barrier traversal (Equation 10) between different regimes of constraint qualification – suggesting that only the method described in this work has reached the global optimum instead of getting stuck in a local level.

6. Conclusion and future work

Integrating generalized geometric constraint qualifications, among which are generalized Mangasarian-Fromovitz constraint qualification (MFCQ) and metric subregularity, with minimax duality and Morse-theoretic landscape analysis creates a general framework for global solution characterization in solving non-convex nonlinear engineering optimization problems. The proposed approach combines constraints stabilizing characteristics and worst-case optimization principles to overcome the drawbacks of traditional approaches to local optimization. It is shown that performance evaluation based on a multi-member truss example demonstrates that the proposed methodology provides the bounded behavior of multipliers and convergence to the global solution. The proposed formulation has an advantage over the standard LICQ-based formulation in terms of avoiding multiplier divergence and convergence to the wrong local traps.

Further research could expand the deterministic landscape-topology solution method to stochastic optimization

environment due to the introduction of the probability measures in the design space. This requires changing from fixed critical points to the stochastic distributions of the stationary states and creating stochastic Morse representations of complicated landscapes. It should be noted that the landscape-topology approach could be applied to infinite-dimensional variational systems, such as distributed parameter engineering models, and therefore tangent and critical cone should be reformulated in a relevant Banach or Hilbert space. It is crucial to investigate the prospect of establishing metric subregularity under weak kinds of topologies to ensure stability and convergence, as well as global solution characterization for advanced engineering optimization problems.

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