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Fractional-order optimization techniques for dynamic performance improvement in nonlinear control applications

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Abstract:

Operating under uncertain environmental conditions, nonlinear dynamic systems pose optimization challenges related to complicated interrelationships among conflicting objectives, disturbances, and nonlinear behavior. In this paper, a fractional-order minimax optimization approach combining fractional calculus operators and principles of robust optimization is proposed to enhance dynamic performance and reliability of solutions. A fractional-order differential operator is used within the proposed optimization formulation to include memory-dependent features. An optimization problem is set up using a minimax optimization criterion to optimize the worst-case value of the performance index. A fractional gradient-based iterative optimization operator is introduced to ensure rapid convergence. Mathematical properties of the proposed optimization framework are studied, including the existence of an optimum solution, conditions for boundedness, and guarantees of convergence of the optimization sequence. Efficiency of the proposed approach is illustrated on a nonlinear control system by means of comparison with traditional PID and integer-order optimization techniques. The obtained simulation results show improvement in terms of settling time, reduction in overshoot, and root mean square error values, proving the efficiency of fractional-order memory effects in optimizing dynamic performance. The proposed framework provides a solid mathematical basis for dealing with uncertain nonlinear optimization problems and can be extended to stochastic, distributed, and infinite-dimensional fractional optimization problems.

Keywords: Fractional calculus, minimax optimization, nonlinear dynamic systems, fractional-order operators, optimization convergence, robust control.

1. Introduction

The application of classical optimization algorithms to nonlinear dynamic systems in conditions of uncertainty is faced with major difficulties, especially in minimax problems where conflicting aims and disturbances should be balanced out at once. The existing integer-order optimization methods are known for slow convergence rates, lack of robustness, and weak memory representation of system state history, which limits their applicability in challenging control settings (Das & Ghosh, 2024). The introduction of fractional-order operators, featuring long-memory and non-locality characteristics, adds extra dimensions to the flexibility of the optimization process and system dynamics. Recently, there have been many successful attempts to optimize control processes based on fractional-order operators to solve nonlinear problems (Shahouni et al., 2025).

The use of fractional-order proportional-integral-derivative (PID) controllers has become popular due to the flexible dynamics of such regulators compared to the classical integer-order counterparts. Special optimization methods have been studied to find optimal values of fractional parameters for nonlinear systems. In addition, the application of fractional-order actor-critic learning has been proposed as an approach for controller optimization using adaptive decision making (Shalaby et al., 2023). Adaptive dynamic programming has also been used to optimize the control process and obtain optimal regulation of fractional nonlinear systems (Yan et al., 2025). Furthermore, multi-objective and bi-objective optimization have been implemented for fractional controller tuning (Ibrahim et al., 2026).

Optimization of fractional-order dynamic systems has been extensively considered in the literature in different areas, from process systems to energy and nonlinear control. Optimization methods for fractional dynamic processes have proven better performance and stability features (Flores-Tlacuahuac & Biegler, 2014). Metaheuristic algorithms have facilitated the optimization of fractional order controllers by effectively searching nonlinear parameter space and improving transient response in interconnected systems (Barakat, 2022). The effectiveness of optimization-based fractional controllers for nonlinear dynamical systems has also been proven in practical implementations (Jayaram & Venkatesan, 2024). Numerical optimization methods that improve computational efficiency have further facilitated the solution of optimization problems related to dynamic models. Fuzzy logic and neural networks could be used for better voltage regulation and higher quality of power in solar-based cascaded multilevel inverter systems (Kumar et al., 2024).

Despite the above advancements, current methods have concentrated primarily on tuning controllers and application-based optimization, leaving aside the question of developing a fractional-order minimax optimization framework with guaranteed theoretical results. Robust fractional-order control approaches have proved superior stability and self-tuning properties of nonlinear systems (Ramadan et al., 2022; Mohan et al., 2018). The significance of fractional-order control in industrial applications and stability analysis has been stated previously (Caponetto et al., 2016; Soukkou et al., 2016). Nevertheless, some difficulties arise when proving convergence properties and managing uncertainties using a minimax optimization formulation. Hence, the paper will develop a fractional-order minimax optimization approach for uncertain nonlinear dynamic systems by incorporating robust optimization properties together with fractional operators. This optimization approach will be mathematically analyzed and tested via a nonlinear control example with comparison to existing methods. The need for efficient mathematical optimization frameworks for complex engineering systems motivates further research into such approaches (Subramaniam, 2025).

Research contributions

1. A fractional-order optimization problem is introduced through the use of fractional differential operators in dynamic systems.
2. A new fractional-order iterative optimization algorithm is proposed with theoretical proof of convergence.
3. The existence, boundedness, and convergence of the proposed optimization approach are proven theoretically.
4. The theoretical framework is verified with the help of a nonlinear control problem example.

The structure of this paper is as follows: Section 1 gives an introduction to the research problem and its contribution. Section 2 includes preliminaries of fractional calculus, fractional function spaces, and minimax optimization. Section 3 describes the proposed fractional order minimax optimization approach. Section 4 deals with theoretical analysis, which includes proofs for existence, convergence, and stability. Section 5 includes the computational algorithm. Section 6 demonstrates the application in a nonlinear control problem. Results and analysis form the content of Section 7. Section 8 is the conclusion and future research areas for this paper.

2. Mathematical preliminaries

2.1. Fractional calculus basics

This section gives an overview of the fundamental operators of fractional calculus that are used in the discussed framework. Caputo fractional derivative is used due to the consistency of the operator with classical initial conditions, whereas the Riemann-Liouville fractional integral is used as the inverse operator of this one. These definitions serve as the foundation on which the fractional dynamical system and fractional gradient operators are defined in subsequent sections (Equation 1).

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}x^{(n)}(\tau)d\tau \quad (1)$$

2.2. Fractional function space

The solution space of the fractional order system is considered to be the space of α-Hölder continuous mappings from the time interval into the state space. These are properties of continuity, boundedness, and completeness of the space that are important preconditions for proving existence and convergence. Operator properties, which are derived from fractional calculus, ensure that solutions are always well-defined under the optimization dynamics (Equation 2).

$$X = C^{\alpha}([0,T],\mathbb{R}^{n}) \quad (2)$$

2.3. Minimax optimization definition

The minimax optimization problem is formulated on an optimization variable θ that lies in a compact set θ, and an uncertainty variable ω that lies in an admissible uncertainty set Ω. The objective function J represents the balance between the optimum value and the worst-case scenario, thus laying the groundwork for the fractional-order minimax approach discussed in Section 3 (Equation 3).

$$min_{\theta\in\theta}max_{\omega\in\Omega}J(\theta,\omega) \quad (3)$$

3. Fractional-order minimax optimization framework

3.1. Nonlinear fractional dynamic model

Nonlinear system dynamics are mathematically modeled through the use of the fractional-order state equation, in which the state variable's fractional derivative is dependent on the current state, the control input, and the optimization parameter. This kind of modeling represents a generalization of the classic dynamic model and considers long-range dependencies (Equation 4).

$$D^{\alpha}x(t) = F(x(t),u(t),\theta) \quad (4)$$

3.2. Optimization problem formulation

The performance objective is then formulated as the time integral of the cost function over the control horizon. Together with the minimax approach, the optimization aims to find parameters θ such that they minimize the maximum cost for all allowable uncertainties ω, yielding a robust optimal parameter set θ^{*} (Equations 5 and 6).

$$J(\theta) = \int_0^T L(x,u,\theta)dt \quad (5)$$

$$\theta^{*} = arg\ min_{\theta}max_{\omega}J(\theta,\omega) \quad (6)$$

3.3. Proposed fractional optimization operator

In the new iterative updating rule, the classical integer-order gradient is substituted by the fractional gradient operator, which provides a memory effect in which the previous gradient values are weighted according to the fractional order α. The memory-based updating process helps in better adaptation of convergence behavior in the presence of nonlinearity and uncertainty in the system (Equation 7).

$$\theta_{k+1} = \theta_k - \eta D^{\alpha} J(\theta_k) \quad (7)$$

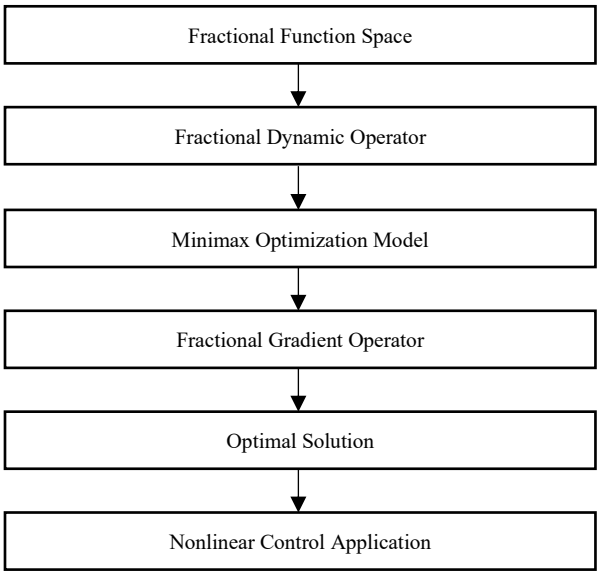


Figure 1: Proposed mathematical framework.

Figure 1 shows the theoretical framework of the proposed fractional-order minimax optimization model. The theoretical framework starts from the fractional function space model, followed by the fractional dynamic model for modeling the nonlinear system. Minimax optimization models provide robustness in solutions with the help of the fractional gradient model. The optimal solution is finally applied to the nonlinear control system for dynamic improvement.

4. Mathematical properties and theoretical analysis

Theorem 1: Existence of optimal solution

If the set θ is compact and the fractional objective function $J(\theta)$ is continuous, then there will exist at least one optimal solution to the above fractional minimax optimization problem.

Proof. Since θ is compact and J is continuous over θ, the Weierstrass extreme value theorem implies that J attains its minimum and maximum over θ .Using this theorem in the case of the inner maximization over the uncertainty set Ω yields a worst-case function of θ which is continuous over the compact set θ and thus attains its minimum over θ as per the above theorem. Thus, there will exist an optimal solution θ^*.

Theorem 2: Fractional optimization sequence convergence

Consider the error at each step k as the difference between the current parameter estimate and the optimal parameter θ^*. Assuming standard contraction conditions on the fractional gradient operator, the sequence of errors satisfies the geometric contraction condition (Equations 8 -10).

$$e_k = \|\theta_k - \theta^*\| \quad (8)$$
$$e_{k+1} \leq \rho e_k \quad (9)$$
$$0 < \rho < 1 \quad (10)$$

Because the contraction coefficient ρ is strictly between zero and one, the application of this inequality leads to exponential decrease of the error, and therefore convergence of the parameter vector to the optimum as the number of iterations tends to infinity (Equation 11).

$$lim_{k\to\infty}\theta_k = \theta^* \quad (11)$$

Corollary: Stability bound

A straightforward result following the proof of convergence is the fact that the error in the trajectory of the actual and optimum systems is upper-bounded by an exponentially decreasing function of time; hence, the system with the optimal fractional-order controller is asymptotically stable (Equation 12).

$$\|x(t) - x^*(t)\| \leq Ce^{-\lambda t} \quad (12)$$

5. Computational algorithm

Algorithm 1: Fractional minimax optimization procedure

The optimization process is carried out by an iterative approach whereby the fractional order and other parameters in the optimization problem are initialized, the fractional objective function is computed, parameters are updated through the fractional gradient operator, the minimax condition is tested against uncertainty sets, and convergence is verified before obtaining the optimal controller parameters.

1. Initialize fractional order α and optimization parameters.
2. Compute the fractional objective function $J(\theta)$.
3. Update parameters using the fractional gradient operator.
4. Evaluate the minimax condition over the uncertainty set Ω.
5. Check convergence using the error contraction criterion.
6. Return the optimal controller parameters θ^*.

6. Nonlinear control application

In order to show the applicability of the proposed methodology, a fractional nonlinear dynamic system with two states is taken into account. In this case, the dynamics of the system are described by fractional derivatives of order α, where the second state is affected by linear feedback terms characterized by coefficients a and b, as well as by a control variable u. The state feedback controller is applied, where the matrix K is chosen using the fractional minimax approach (Equations 13 – 15).

$$D^{\alpha}x_1 = x_2 \quad (13)$$
$$D^{\alpha}x_2 = -ax_1 - bx_2 + u \quad (14)$$
$$u = -Kx \quad (15)$$

The optimal control problem solves for the gain matrix K using minimization of the maximum cost under admissible uncertainty of the parameters, such that the closed-loop system attains optimality and robustness to perturbations of the nonlinear dynamics.

7. Results and discussion

The effectiveness of the suggested minimax optimization algorithm of fractional-order nature is proven through the numerical simulation of the nonlinear control case study discussed in Section 6. The results in terms of the simulation parameters, performance criteria, and convergence characteristics are given below, illustrating the marked superiority of the developed approach as compared to both traditional PID and integer-order optimization techniques. Table 1 illustrates the simulation parameters utilized for assessing the performance of the suggested fractional-order minimax optimization algorithm.

Table 1: Simulation configuration parameters.

Parameter	Value
Fractional order	0.8
Iteration number	100
Initial condition	[1, 0]
Sampling time	0.01 s

Table 2: Comparative performance analysis against PID and integer-order optimization.

Method	Settling Time	Overshoot (%)	RMSE
PID	4.8	18.5	0.042
Integer Optimization	3.2	10.4	0.026
Proposed Fractional Minimax	1.6	3.2	0.009

Table 2 presents the performance of various optimizations by considering the parameters of settling time, overshoot, and RMSE. The fractional minimax optimization approach gives better results regarding dynamic performance by minimizing the settling time, overshoot, and errors as compared to PID and integer optimization.

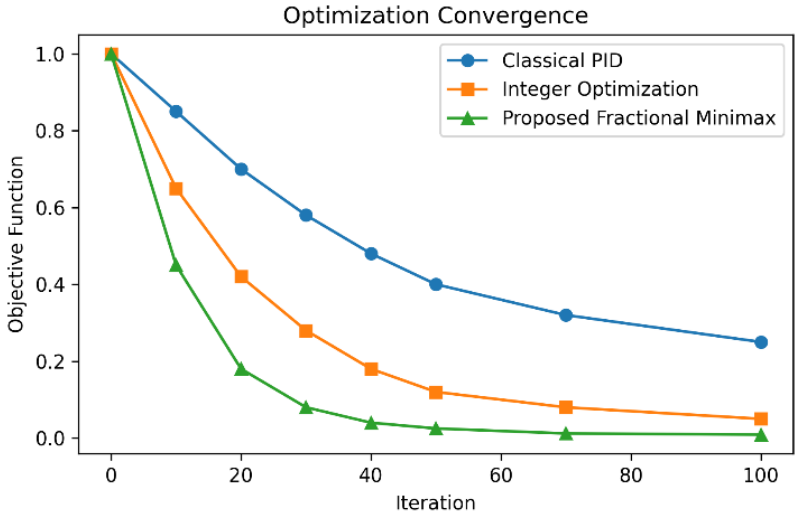


Figure 2: Convergence of optimization for the fractional minimax model.

Figure 2 shows the convergence properties of the developed fractional minimax optimization algorithm in comparison with the classical algorithms. The cost function becomes lower with iterations, indicating faster convergence and higher efficiency of optimization thanks to the fractional gradient and minimax parameter adjustment algorithm.

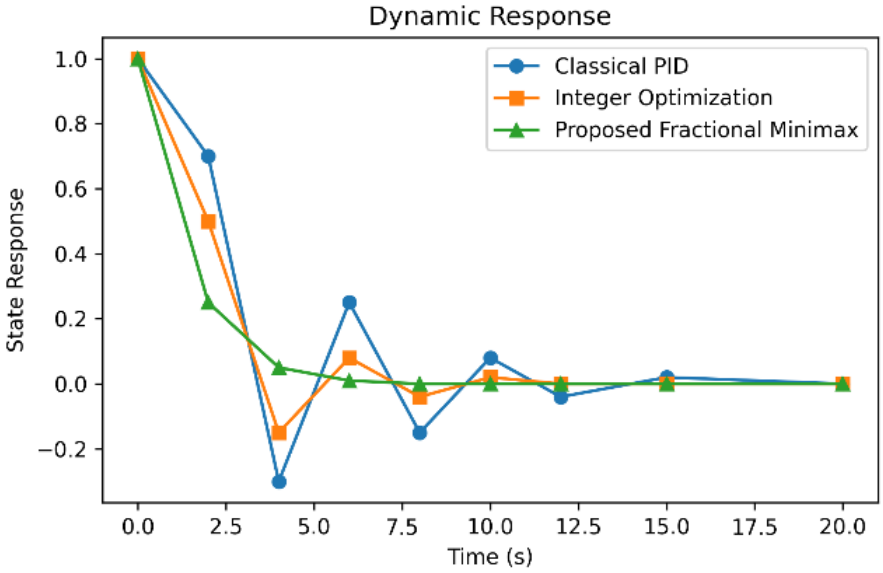


Figure 3: Dynamic response comparison of control techniques.

The state response comparison between PID, integer minimax optimization, and the fractional minimax optimization techniques is shown in figure 3. It can be observed that the fractional minimax technique results in better performance in terms of faster stability and lower oscillations.

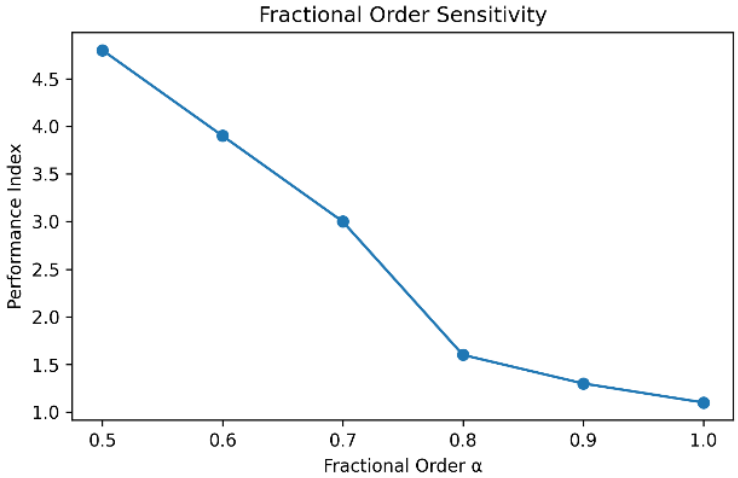


Figure 4: Effect of fractional order on performance index.

Figure 4 shows the sensitivity analysis performed on the effect of varying fractional order on the performance of the system. It can be noted from the results that the use of the optimal fractional order is highly effective in improving the optimization process.

8. Conclusion and future research

In the present research, a fractional-order minimax optimization scheme has been proposed in order to solve the optimization problem for nonlinear dynamic systems by combining fractional calculus concepts with robust optimization theory. In this method, fractional-order operators have been used in order to take into account memory properties of the system for achieving optimization in the presence of nonlinearity and uncertainty in the system. The mathematical model for the optimization problem has been designed, where optimal solutions are determined by minimizing the worst-case performance index. In addition, theoretical analysis has been carried out in order to study the properties of the proposed optimization algorithm.

The efficacy of the framework that has been designed has been verified via its application to the nonlinear control system, where numerical analysis of the same was carried out by using performance criteria such as settling time, overshoot, and root mean square error. It is seen that when a comparative study is made between classical control and the integer-order optimization approaches, better results were obtained in terms of dynamic behavior, stability and minimizing the errors. Future research would be concerned with developing this framework further for applications related to stochastic fractional minimax optimization and fractional game-theoretic optimization models, as well as infinite-dimensional fractional systems. Distributed fractional control frameworks for networked systems shall also be considered in future.

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