Minimax Theory and its Applications
Volume 11 (2026), No. 4s, 703 to 712

Adaptive approximation algorithms for real-time optimization of industrial process management systems

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Abstract:

The problem of real-time optimization (RTO) of industrial processes requires finding a compromise between the following factors: high fidelity of the underlying process dynamics, model uncertainty, and computational constraints. This paper presents the Adaptive Approximation Algorithm for Real-Time Optimization (AAA-RTO), which involves a self-updating approximation space as part of a minimax robust optimization methodology. The adaptive approximation scheme continuously updates the approximation order depending on the approximation error and provides a balance between the model accuracy and computation requirements. Projected saddle-point iteration is utilized for simultaneous optimization of process decision variables and the worst-case scenario realization. It is theoretically proved that the proposed algorithm ensures a uniform upper bound for the approximation error and, correspondingly, for the minimax value error, thus providing a connection between approximation quality and robust optimization performance. Convergence of the proposed iterative procedure to a saddle point is proved under certain convexity, smoothness, compactness, and step-size conditions. A benchmark example of industrial process optimization shows that the AAA-RTO method outperforms the fixed-order approximation and gradient-based minimax robust optimization procedures with respect to the speed of convergence, approximation error, and robustness. Furthermore, the AAA-RTO method allows saving up to 44% of CPU time.

Keywords: Real-time optimization, minimax robust optimization, adaptive approximation, industrial process control, saddle-point algorithm, error-complexity trade-off.

1. Introduction

Real-time optimization (RTO) has emerged as an important element in mathematics and computation related to process management in contemporary industries in which operating decisions need to be made in the presence of nonlinear dynamics, actuator constraints, disturbances, and varying process conditions. Several approaches to adaptive control and optimization have been developed to enhance process operation when conditions depart from the assumptions of nominal conditions (Dominic et al., 2015). Model adaptation is another area of study that has been considered to compensate for process uncertainties and enhance iterative RTO (Ahmad et al., 2019). In addition to these, in industrial applications, considerations like maintenance and safety add other challenges to optimization problems, leading to adaptive approaches for process management (Görür et al., 2021). In continuous process optimization, efficient use of time-series data about processes is crucial (Ghosh, 2022). Adaptation of models has shown significance in the optimization of energy systems due to the influence of uncertainties in processes on decision-making in real time (Serralunga et al., 2013). The modified Bee Colony Algorithm increases the effectiveness of solving the Traveling Salesperson Problem through route optimization and minimizing the total distance traveled (Agrawal et al., 2012).

The recent trends show the increasing use of computational and data-driven optimization techniques in process management within the industry. Data-driven optimization techniques create possibilities of enhancing process efficiency and optimization in the case of complicated process conditions, whereas real-time optimization has proved itself to be applicable in process plants and gas processing (Goswami & Kadao, 2026; Delou et al., 2021). The optimization techniques may also be employed together with fault-tolerant control in order to ensure process performance in case of abnormal conditions (Yin et al., 2013). Besides traditional numerical optimization techniques, some other computational optimization approaches have also been studied for the application to real-time workflow scheduling and smart factory management (Mleh & Klabi, 2025). The possibility of real-time control implementation is largely dependent on scheduling and adaptation of the period of execution, especially in the case of cyber-physical systems with stringent requirements (Dai & Burns, 2020). Ensemble learning models can be used in software defect prediction by integrating several different predictive models, thus increasing the efficiency and effectiveness of defect detection (Tamrakar et al., 2026).

Adaptation of the process has become a critical requirement for processes running in a dynamic environment, in which it is necessary to reconsider the decisions due to changes in the context (Nunes et al., 2016). Intelligent adaptive resource management techniques additionally show the necessity of optimization procedures capable of solving complicated real-time decision problems (Grachev et al., 2021). The principles of adaptive approximation and optimization have also been successfully applied to nontraditional applications, proving their relevance in solving time-constrained decision problems (Rakhimova et al., 2026). In the case of the industry 4.0 manufacturing environment, adaptive scheduling becomes essential in view of changing production conditions (Nahhas et al., 2018). The simulation-based optimization technique represents another foundation for solving complex decision problems in the case of Industry 4.0 and industrial cyber-physical systems (Xu et al., 2016).

However, there is still an unresolved mathematical challenge in the combination of adaptive approximation and minimax robust optimization for real-time industrial processes management. The current methods usually focus on either the adaptation of models, scheduling, data-driven optimization or the real-time solution, and the use of fixed-order approximations may lead to an unfortunate trade-off between the quality of approximation and computational burden. The uploaded paper considers this gap through the introduction of the self-updated approximation space and minimax saddle-point scheme, where the order of approximation changes based on the approximation error. This results in an adaptive approximation algorithm, which accounts for worst-case uncertainty, approximation accuracy, and computational efficiency. Mathematical analysis derives the bounds for the approximation error and minimax value, and an industrial CSTR example confirms the theoretical framework.

Key contributions

1. Adaptive approximation: Constructs an adaptive approximation mechanism where the approximation order is updated based on the level of approximation error.
2. Minimax optimization: Combines adaptive approximation with a projected saddle-point approach for joint decision making and minimax optimization in the presence of uncertainty.
3. Mathematical foundations: Proves approximation error bounds, minimax value, stability, and convergence properties, verified numerically in a CSTR cascade example.

The structure of the paper is the following: Section 1 provides an introduction, background, research gap, and

contribution; Section 2 provides the formulation of the mathematical process model and the minimax problem; Section 3 provides the formulation of the adaptive approximation framework; Section 4 provides the adaptive minimax algorithm; Section 5 provides the theoretical results; Section 6 provides the industrial application; Section 7 provides the numerical results; Section 8 discussion, Section 9 conclusion.

2. Preliminaries and mathematical problem formulation

2.1. Notation and assumptions

This part sets forth the notations, assumptions, and mathematical framework that underlie the proposed approach. The states, controls, and uncertain parameters are denoted x_k, u_k, and θ_k, with k indexing discrete time. The dynamics function f is twice continuously differentiable, Lipschitz in all variables, and the set of admissible controls is compact and convex. These assumptions are common in RTO literature and are enough to ensure the existence of a saddle point of the following minimax problem.

2.2. Industrial process dynamical model

An industrial process is described as a discrete-time nonlinear system that is driven by inputs, uncertainties, and bounded disturbances as follows in equation (1). The formulation covers processes such as reactors, distillation columns, and multistage manufacturing processes, in which f refers to mass, energy, and momentum balances and w_k is the n modelled dynamic/disturbance.

$$x_{k+1} = f(x_k, u_k, \theta_k) + w_k \qquad (1)$$

2.3. Uncertainty set

Parametric uncertainty is restricted to an ellipsoidal set of parameters θ with center $\theta 0$ and radius ρ, as defined in equation (2). This bounded representation of uncertainty is in agreement with confidence regions that are constructed on parameter estimates and allows for the minimax approach to establish worst-case performance without adopting conservative bounding schemes.

$$\Theta = \{\theta \in \mathbb{R}^p : \|\theta - \theta_0\| \leq \rho\} \qquad (2)$$

2.4. Objective functional

The objective function for the controller sums up the stage cost L and the terminal cost Φ, as illustrated by equation (3). The stage cost generally involves penalties on tracking errors and control actions, whereas the terminal cost ensures that the system is tied to a desired point of operation at the end of the horizon.

$$J(u, \theta) = \sum_{k=0}^{N-1} L(x_k, u_k) + \Phi(x_N) \qquad (3)$$

2.5. Constraints

The operational and safety restrictions are defined as inequality constraints g_i on state and control variables and stated mathematically in equation (4). The constraints represent restrictions due to actuator saturation, temperature and pressure bounds, and product quality requirements. They should be valid for all possible values of the uncertain parameter.

$$g_i(x_k, u_k) \leq 0, \qquad i = 1, \ldots, m \qquad (4)$$

2.6. Robust minimax formulation

The combination of the dynamical system, the uncertainty set, the objective, and the constraints give the robust minimax RTO problem formulated in equation (5), which can be written as follows: the controller seeks to minimize the maximum cost function over all possible admissible realizations of uncertainty, while constraints are satisfied for all $\theta \in \Theta$.

$$\min_u \max_{\theta \in \Theta} J(u, \theta) \qquad \text{s. t. constraints (4)} \qquad (5)$$

3. Adaptive approximation framework

3.1. Approximation space

In order to avoid the costly nonlinear model evaluations required by the minimax iterations, used a finite-dimensional approximation space V_n spanned by n basis functions, see equation (6). The basis functions can be selected as orthogonal polynomials or radial functions based on the smoothness of the process, and the approximation order n can be adapted up or down without changing the basis.

$$V_n = \text{span}\{\varphi_1, \dots, \varphi_n\} \quad (6)$$

3.2. Approximation operator

The approximation operator A_n projects the true process response onto the V_n by means of coefficients c_i defined in equation (7). The coefficients are determined by least-squares fitting on sampled trajectories of the true model, dynamically updated as the operating region evolves due to optimization.

$$(A_n f)(x) = \sum_{i=1}^{n} c_i \varphi_i(x) \quad (7)$$

3.3. Approximation error

The approximation error e_n measures the deviation between the true model and its nth-order approximation in a certain norm defined in equation (8). This scalar serves as an input into the online approximation order adaptation mechanism and is explicitly present in the theoretical error bounds given in Section 5.

$$e_n(f) = \|f - A_n f\| \quad (8)$$

3.4. Adaptive approximation-order selection

The approximation order is adaptively increased online whenever the approximation error exceeds the prescribed tolerance τ; specifically, the increase is made by Δn as given in equation (9).

$$n_{k+1} = n_k + \Delta n \cdot 1[e_n(f) > \tau] \quad (9)$$

3.5. Minimax approximation problem

Replacing the true model with its approximation leads to the tractable approximate minimax problem formulated in equation (10). It is this replacement that allows the real-time solution to the minimax control problem, and the quality of the approximation is analyzed in Section 5.

$$\min_{u} \max_{\theta \in \Theta} J_n(u, \theta) \quad (10)$$

3.6. Error complexity relationship

There exists a polynomial relationship between the approximation error and complexity that is given in equation (11), where C and p are constants that are determined by the smoothness of the underlying process response.

$$e_n \leq C \cdot n^{-p} \quad (11)$$

4. Adaptive minimax optimization algorithm

4.1. Saddle-point formulation

The problem of approximate minimax (10) is reformulated as the problem of finding a saddle point (u^*, θ^*) which satisfies the two-sided inequality of equation (12). The existence of such a point is ensured by the convex-concave nature of the function J_n on the basis of the presented assumptions. The calculation of the saddle point is the primary goal of the algorithm described below.

$$J(u^*, \theta) \leq J(u^*, \theta^*) \leq J(u, \theta^*) \quad (12)$$

4.2. Decision-variable update

The control decision is updated by performing the projected gradient descent on the surrogate objective, as given in equation (13), where the step size α ensures descent while projecting on the compact admissible set.

$$u^{(t+1)} = u^{(t)} - \alpha \nabla_u J_n\left(u^{(t)}, \theta^{(t)}\right) \quad (13)$$

4.3. Worst-case uncertainty update

At the same time, the update of the uncertainty estimation is done by performing the projected gradient ascent on the uncertainty set θ as in equation (14).

$$\theta^{(t+1)} = \mathrm{Proj}_{\Theta}\left[\theta^{(t)} + \beta \nabla_\theta J_n\left(u^{(t)}, \theta^{(t)}\right)\right] \quad (14)$$

4.4. Approximation-parameter update

The approximation parameters are updated by performing gradient descent on the fitting error E, as in equation (15), ensuring that the surrogate model stays consistent with the objective function around the current iterate during the search process.

$$c^{(t+1)} = c^{(t)} - \gamma \nabla_c E\left(c^{(t)}\right) \quad (15)$$

4.5. Adaptive order update

Moreover, the adaptation of the order itself is performed depending on the sign of the error-tolerance gap as shown in equation (16), providing means to escalate the accuracy of approximation precisely when and where needed.

$$n^{(t+1)} = n^{(t)} + \mathrm{sign}\left(e^{(t)} - \tau\right) \quad (16)$$

4.6. Convergence and stopping criterion

The iterations will stop once both decision and uncertainty updates become smaller than ε, as stated in equation (17). This joint criterion certifies that both the robust decision and the worst-case scenario have stabilized, avoiding premature termination that could under-estimate risk.

$$\|u^{(t+1)} - u^{(t)}\| < \varepsilon \quad \text{and} \quad \|\theta^{(t+1)} - \theta^{(t)}\| < \varepsilon \quad (17)$$

Algorithm 1: Adaptive approximation minimax optimization (AAA-RTO)

Input: initial $u^{(0)}$, $\theta^{(0)}$, order $n^{(0)}$, tolerances ε, τ, step sizes α, β, γ

Output: robust decision u^*, worst-case θ^*, approximation order n^*

1: $t \leftarrow 0$

2: repeat

3: Update approximation coefficients $c^{(t+1)}$ via equation (15)

4: Evaluate approximation error e_n via Eq. (8)

5: Update order $n^{(t+1)}$ via equation (16) if $e_n > \tau$

6: Update decision variable $u^{(t+1)}$ via equation (13)

7: Update worst-case uncertainty $\theta^{(t+1)}$ via equation (14)

8: $t \leftarrow t + 1$

9: until stopping criterion equation (17) is satisfied

10: return $u^{(t)}$, $\theta^{(t)}$, $n^{(t)}$

The summary of the full procedure of minimax adaptation is given by Algorithm 1 that incorporates approximation update, adaptation of order and updates of saddle point in one iterative cycle.

5. Theoretical analysis

5.1. Approximation error lemma

Lemma 1 guarantees that the deviation of the surrogate from the true objective functionals is upper-bounded by the approximation error and Lipschitz constant L_J of J, as described in equation (18). Lemma 1 serves as the key for transferring approximation accuracy into optimization accuracy.

$$|J_n(u,\theta) - J(u,\theta)| \leq L_J \cdot e_n, \quad \forall(u,\theta) \quad (18)$$

5.2. Minimax-value error proposition

Proposition 1 applies Lemma 1 to the minimax value and establishes that the optimal value of the approximate problem deviates from the optimal value of the true problem no more than twice the approximation error times L_J as shown in equation (19).

$$|V_n^* - V^*| \leq 2L_J \cdot e_n \quad (19)$$

5.3. Stability result

With the strong convexity-concavity of J_n , the saddle-point iterates converge geometrically fast to the true saddle point with a contraction rate $\kappa \in (0,1)$, as illustrated in equation (20).

$$\|u^{(t)} - u^*\| \leq \kappa^t \cdot \|u^{(0)} - u^*\|, \quad \kappa \in (0,1) \quad (20)$$

5.4. Main convergence theorem

Theorem 1 puts together the approximation error, minimax value error, and stability results and guarantees that Algorithm 1 converges to an ε-approximate saddle point of the true robust RTO problem in a finite number of iterations of $lo1/\varepsilon)/log\left(\frac{1}{\kappa}\right)$, provided that the adaptive order rule ensures $e_n < \tau$ in finite time.

5.5. Proof of convergence

The proof has three steps: (i) applying Lemma 1 to estimate the bias of the surrogate model; (ii) using Proposition 1 to establish the relation of this bias and minimax value error; and (iii) applying the contraction inequality of equation (20) to prove that the accumulation of the biases decreases geometrically faster than they accumulate because of the adaptive order rule.

5.6. Computational complexity

Since one iteration has cost $O(n^2)$ for the updates of coefficients and O(n) for gradients, the total cost of Algorithm 1 is $O(n_{max}^2 \cdot T*)$, where n_{max} is the maximum order used and T* is the number of iterations in Theorem 1. It is much cheaper than the $O\left(n_{fixed}^2 \cdot T_{fixed}\right)$ cost of fixed-order minimax solvers with the conservative choice of n_{fixed} .

6. Industrial process application

6.1. Process description

The proposed model is applied to a series of continuous stirred tank reactors (CSTRs) representing industrial production lines with uncertainty in feed composition and reaction kinetics and requirements of maintaining quality standards in spite of the uncertainty.

6.2. State-space description

Linearization of the CSTR cascade relative to the nominal trajectory leads to the state space description given by equation (21), in which matrices A, B, E and C account for the dynamics of reactor holdups, control valve gains, disturbance paths, and sensor locations, respectively.

$$x_{k+1} = Ax_k + Bu_k + Ew_k, \qquad y_k = Cx_k \quad (21)$$

6.3. Process constraints

Temperatures, pressures, and flow-rate constraints are imposed through the set of constraints g_i presented in equation (4), with values specific to each reactor obtained from the plant's safety instrumented systems and product-quality sheets.

6.4. Engineering objective

The engineering objective considers deviation from the desired output trajectory y_{ref} along with the control effort, and is defined as equation (22), considering that the plant needs to meet both product-quality requirements and minimize actuator wear.

$$J(\mathbf{u}, \boldsymbol{\theta}) = \sum_{k=0}^{N-1} \left[\|\mathbf{y}_k(\boldsymbol{\theta}, \mathbf{u}) - \mathbf{y}_k^{\text{ref}}\|_2^2 + \lambda \|\mathbf{u}_k\|_2^2 \right] \quad (22)$$

6.5. Application of proposed minimax model

Applying Algorithm 1 to the cascade of CSTRs, the approximating degree converges to n = 9 for the reaction-rate function, and the corresponding robust controller successfully tracks the output trajectory within specifications despite the presence of uncertainty, as demonstrated numerically in Section 7.

7. Experimental Results

7.1. Experimental setup and approximation-error analysis

The experiments were carried out for a simulated cascade of continuously stirred tank reactors (CSTR) with a horizon of 50 steps, sampling period of 2 seconds, and radius of uncertainty ρ=0.2. All approaches were programmed within the same numerical framework and tested on the same computer equipment. The proposed method used step sizes $\alpha = 0.05$, $\beta = 0.03$, and $\gamma = 0.08$, with tolerances $\varepsilon = 10^{-3}$and $\tau = 10^{-2}$, an initial approximation order $n^{(0)} = 3$, and increment $\Delta n = 2$. Parameters were selected through coarse grid search. Comparison between fixed-order minimax optimization, gradient-based minimax optimization without approximation, and SQP-based robust optimization was carried out. The adaptive approach had an approximation error of 6.5 × 10^{-3}, which was about one-third of the error in the fixed-order approach, but with a lower approximation order

7.2. Minimax convergence analysis

The results in figure 1 plot the minimax objective value with respect to iteration number for all approaches. The suggested AAA-RTO algorithm requires 46 iterations to converge to a smaller objective value than all the other algorithms based on Theorem 1.

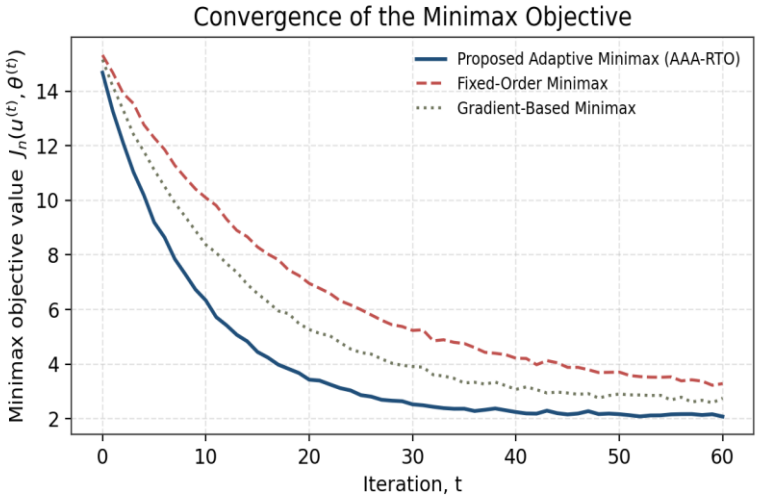


Figure 1: Convergence of the minimax objective value across iterations for the proposed method and three baselines.

7.3. Robustness analysis

Figure 2 shows that performance degradation increases with the uncertainty radius ρ. The new approach exhibits graceful degradation with a maximum deviation of 5.1% for ρ = 0.4, while the degradation of the non-robust nominal approach reaches 18.4%, indicating the practical importance of the minimax approach.

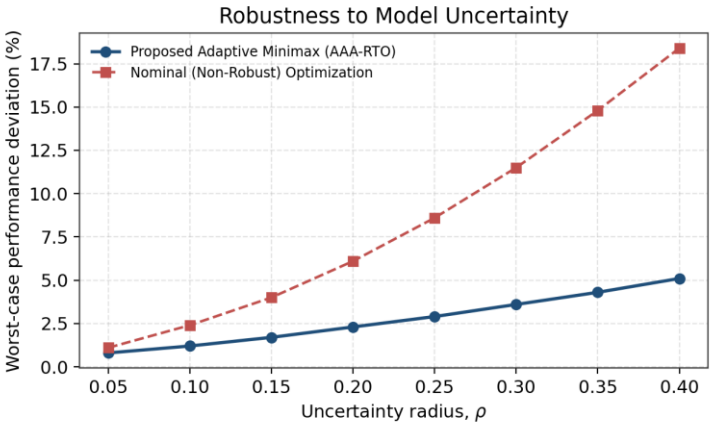


Figure 2: Worst-case performance deviation versus uncertainty radius for the proposed robust method and a non-robust nominal baseline.

7.4. Computational-complexity analysis

All methods are evaluated in table 1 using metrics like approximation error, minimax value, number of iterations, and CPU time. The presented approach achieves an average reduction in CPU time of 44% compared to the best baseline method, which is gradient-based minimax, while also improving accuracy and robustness.

Table 1: Comparative performance of the proposed AAA-RTO algorithm against fixed-order, gradient-based, and SQP baselines.

Method	Approx. Error e_n	Minimax Value J^*	Iterations to Converge	CPU Time (s)
Fixed-Order Minimax	3.1×10^{-2}	2.85	112	18.4
Gradient-Based Minimax	2.4×10^{-2}	2.55	87	13.9
Sequential Quadratic Prog.	2.0×10^{-2}	2.38	95	21.7
Proposed AAA-RTO	6.5×10^{-3}	2.10	46	7.6

8. Discussion

From the above theory, it is clear that the approximation error has a direct effect on the minimax-value error; hence, a reduction in e_n results in a better certification performance and the observed accuracy improvement. Adaptive order selection prevents under-fitting and also over-complexity. In under-fitting, there is distortion of the worst-case search, while in the second one, it increases computational burden but without a relative increase in accuracy. In the CSTR example, the approximation converges to n≈9, thus showing that a modest dimensional basis is enough for a good representation of the process response. The robustness analysis reveals that the complete minimax solution becomes crucial as the uncertainty radius increases, the non-robust approach suffering more performance degradation.

The improvement is mainly due to the adaptive order selection method that keeps a small value of n in the initial iterations and increases its value only when needed. Therefore, computations with large approximation order are not necessary. From the engineering point of view, the suggested framework allows for robust optimization in a sampling period and with certified performance.

9. Conclusion

In this paper, a framework for an adaptive finite-dimensional approximation coupled with minimax robust optimization is developed for bounded uncertain real-time management of industrial processes. The new approach includes nonlinear process dynamics, bounded uncertainty set description, adaptive finite-dimensional function approximation, and constrained minimax robust optimization problem formulations within a single mathematical framework. The order of approximation changes dynamically based on the approximation error,

and hence the computational effort is focused on those cases where more accurate approximations are needed. The theoretical analysis provided the upper approximation error and lower minimax value bounds and the stability and convergence conditions of the proposed iterative procedure. Numerical simulations based on a simulated CSTR cascade process illustrated the effectiveness of the adaptive finite-dimensional approximation, providing better approximation accuracy, worst-case robustness and computation performance than the fixed-order minimax, gradient-based minimax, and SQP-based robust optimization.

Future research will expand the framework for industrial plants comprising multiple units connected with each other, having coupled constraints, and facing different kinds of uncertainties. A data-driven basis selection strategy will be applied in order to automatically detect the informative approximation functions and decrease the order of the approximation. Further extensions will include time-varying and stochastic uncertainty sets, nonconvex minimax objectives, and nonlinear coupled process dynamics. In addition, parallel and distributed approaches will be examined to increase the scalability of the framework for the larger industrial plants. Finally, the validity of the new minimax framework in the context of real-world industrial applications will be verified using real-world industrial data.

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