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Complementarity-based equilibrium computation methods for traffic flow management in intelligent transportation networks

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Abstract:

The intelligent transportation system needs a robust mathematical approach to compute equilibrium in the presence of intricate interactions between traffic flows, congestion effects, and network constraints. In this paper, proposes a novel equilibrium computation methodology based on complementarity, which formulates traffic flow management into a nonlinear complementarity problem and reformulates the problem using a variational inequality formulation. The approach uses monotone operator theory in establishing the equilibrium existence, uniqueness, convergence, and perturbation stability properties under the assumptions of continuity, strong monotonicity, and Lipschitz condition. A merit function is used based on the Fisher-Burmeister function to convert complementarity conditions to a smooth residual minimization problem and develop an iterative algorithm using a projection technique to solve the equilibrium problem efficiently. To provide theoretical results that show that the proposed method has linear convergence to the unique solution of the problem and bounded sensitivity to operator disturbance. Numerical simulations for large-scale transportation network problem instances show the better efficiency of the algorithm than Frank-Wolfe, Gradient Projection, and Extragradient methods in computing the equilibrium point with less equilibrium gap, complementarity residual, and faster convergence speed.

Keywords: Nonlinear complementarity problem, variational inequality, monotone operator theory, traffic equilibrium, projection algorithm, intelligent transportation networks.

1. Introduction

The growing trends of urbanization, connectivity, and intelligent infrastructures for transportation have changed the traffic management process into a complex equilibrium problem where vehicle-route-resource interaction takes place. An intelligent transportation system uses sensor technologies along with computing and communication tools to increase mobility effectiveness, to mitigate congestion, and to enable adaptive decision-making. Within such a network, the route choice of each individual constantly affects the traffic state of all, and there is a need to apply mathematical equilibrium models that capture the relationship between flows and corresponding costs. Formulations using complementarity principles are the most natural way of capturing equilibrium behavior, as the properties of flow admissibility, cost relationships, and user decision-making are defined mathematically instead of being stated in terms of optimization problems (Bagloee et al., 2017). Modern studies of smart traffic management stress the role of advanced computing frameworks in coping with congestion and increasing the performance of urban mobility (Goswami & Kadao, 2025). The analysis of transportation planning systems shows that network topology, land use structure and mobility demand are tightly coupled (Xu & Yan, 2021).

The construction of large-scale intelligent transportation networks poses great difficulties in mathematics due to high-dimensional flow variables, non-linear congestion operators, and complicated constraints regarding network capacity and interactions. Dynamic user equilibrium models show that queue spillbacks, time-varying congestion, and non-linear link interactions may not always be appropriately modeled in standard optimization form (Ma et al., 2018). The appearance of connected vehicles, ridesharing systems, and carpooling platforms raises even more difficulties in terms of equilibrium due to the addition of new decision variables and couplings (Gu & Xv, 2026). Modern transportation systems need integrated mathematical models that can model the interactions of transportation infrastructure with energy infrastructure (Agrawal et al., 2016). Current models for traffic assignments usually make strict assumptions such as symmetry and separability of cost functionals, which are very restrictive when applied in practical transportation settings. While machine learning-based prediction algorithms can make traffic estimates better, they do not provide a theoretical background of equilibrium stability and convergence (Akash et al., 2022). Congestion-aware routing algorithms, although effective in utilizing resources in the network, do not provide mathematical guarantees about equilibrium solutions (Chaitanya Prasad et al., 2024). Ensemble-based vehicle detection and classification highlight the benefits of using an ensemble of various computational algorithms to increase efficiency and accuracy of processing, which applies to distributed optimization approaches in engineering networks (Lv et al., 2024).

Complementarity theory and the variational inequality approach offer a generalized mathematical framework for the computation of the network equilibrium without making use of the strong convexity assumptions and equivalent potential functions. The mathematical models allow for characterization of the equilibrium problem via nonlinear operators and thus can be utilized in analysis of the asymmetric and coupled transportation systems. Modeling of equilibrium problems using the complementarity theory is known to be effective in analysis of equilibrium relations between many variables subject to constraints (Gabriel et al., 2012). Modern transportation optimization approaches that take into account transit-oriented development reveal the impact of the features of the infrastructure and mobility on network equilibrium (Kim & Kim, 2025). Intelligent mobility approaches with autonomous vehicles and adaptive transportation interfaces require utilization of mathematical frameworks to address nonlinearity and large-scale decision-making issues (Zhou et al., 2026). Research on smart city transportation addresses the problem of integrated analysis of the processes of mobility management and optimization (Amini Pishro et al., 2023). Research on urban competitiveness and urban infrastructure assessment demonstrates the necessity of employing generalized modeling techniques to describe complicated relationships between the elements of the transportation system (Abdrabo et al., 2024). Comparing transportation systems of the metropolitan areas reveals that mobility equilibrium is associated with various structural and operational issues, which need advanced computational techniques (Chen et al., 2024). Furthermore, physics-informed computational approaches reveal the importance of incorporating the mathematical constraints and efficient algorithmic techniques for modeling complex urban systems (Zhong et al., 2026). Consequently, this paper develops a complementarity-based framework for equilibrium computation, which allows for rigorous existence, uniqueness, stability, and convergence analysis for large-scale intelligent transportation networks.

Research contributions

This work proposes a complementarity equilibrium computation method for intelligent transport networks that offers: (1) a mathematical modeling of the equilibrium problem using nonlinear complementarity and variational inequalities; (2) reformulating the problem into a merit function based on Fisher-Burmeister to minimize the smooth residual function; (3) a projection algorithm with clear stopping rule; (4) proof of equilibrium existence, uniqueness, and linear convergence under monotonicity conditions, along with bounds on perturbation stability;

and (5) numerical results on network examples comparing with Frank-Wolfe, gradient projection and extragradient methods.

Section 1 outlines the background of the intelligent transportation system equilibrium problem, mathematics, shortcomings of current techniques, and the reasons behind the need to use complementarity-based modeling. Section 2 covers mathematical foundations, including nonlinear complementarity problems, variational inequalities and operators. Section 3 sets up traffic flow equilibrium. Section 4 sets out the suggested computation model. Section 5 covers convergence and stability analysis. Section 6 carries out numerical evaluation. Section 7 offers conclusions, and Section 8 concludes the paper.

2. Mathematical background and problem definition

This section sets out the mathematical groundwork that needs to be established prior to setting out the context of traffic engineering. The mathematical concepts of the nonlinear complementarity problem and variational inequalities are stated first, while the operator theoretic concepts of monotonicity, strong monotonicity, Lipschitz continuity, and coercivity are set out later for establishing the results in Section 5.

2.1. Nonlinear complementarity problem

The nonlinear complementarity problem (NCP) is the search for a vector x where both x and its operator $F(x)$ have to be non-negative at the same time and satisfy the additional constraint that the inner product is zero. This concept extends the optimality criteria of constrained optimization problems to cases where an objective function does not exist. The language of NCPs is thus the appropriate one to express equilibrium conditions in a system with mutual influences among its components (Equation 1).

$$x \geq 0, \quad F(x) \geq 0, \quad x^T F(x) = 0 \qquad (1)$$

2.2. Variational inequality framework

The Variational Inequality (VI) Problem looks for a point x^* in a set of feasible alternatives such that the evaluation of the vector field F at x^*, together with the direction towards any other feasible alternative y, yields a value which is not negative. This defines x^* as an alternative from which any feasible deviation cannot decrease costs, which is exactly the desired property of a traffic equilibrium. Under normal conditions, the VI problem and the NCP problem are equivalent (Equation 2).

$$\langle F(x^*), y - x^* \rangle \geq 0 \qquad (2)$$

2.3. Monotone operator properties

The following four (Equations 3 - 6) properties of operators dictate the analysis to be performed. Monotonicity implies that the inner product between the operator difference and argument difference will always be positive, providing a minimum structural requirement for regularity. Strong monotonicity improves upon this requirement by providing an inequality with a quadratic lower bound on the operator difference with respect to the parameter μ to ensure uniqueness of the solution. Lipschitz continuity puts an upper limit on the rate of change of the operator through a constant L.

$$\langle F(x) - F(y), x - y \rangle \geq 0 \qquad (3)$$

$$\langle F(x) - F(y), x - y \rangle \geq \mu \|x - y\|^2 \qquad (4)$$

$$\|F(x) - F(y)\| \leq L\|x - y\| \qquad (5)$$

$$\langle F(x) - F(y), x - y \rangle \geq \beta \|F(x) - F(y)\|^2 \qquad (6)$$

3. Mathematical formulation of traffic flow equilibrium

After presenting the abstract complementarity and variational inequality formulation, this section discusses the engineering problem of representation of the intelligent transportation network topology, traffic cost operator construction via travel time and external costs, and complementarity equilibrium formulation modeling the rational behavior and traffic flow taking into account congestion effects.

3.1. Representation of the intelligent transportation network

The transportation network is modeled by the directed graph G whose vertex set V corresponds to either intersections or traffic zones, while the edge set E corresponds to road links connecting them. Flow variables associated with each link are non-negative, while each origin-destination pair creates traffic demands allocated on paths passing along the network graph. This graph representation constitutes the backbone for further introduction of the flow vector x, feasible set, and cost operator F (Equation 7).

$$G = (V, E) \qquad (7)$$

3.2. Traffic cost operator construction

The cost operator F in terms of traffic is defined as the summation of two components, $T(x)$, related to travel time, which captures the congestion aspects such as time delay due to traffic volumes, and the constant vector C. The constant vector C contains costs that are independent of the traffic flows and may include tolls, fuel costs, etc. This helps in ensuring that the nonlinear and possibly asymmetrical aspects of the congestion can be captured by the travel time operator $T(x)$(Equation 8).

$$F(x) = T(x) + C \quad (8)$$

3.3. Complementarity-based traffic equilibrium model

Traffic Equilibrium can be stated as a problem of complementarity where the variables, x and $F(x)$, are both positive and orthogonal to each other. The meaning is straightforward - a road/path with positive flow should have minimum, and hence equalized, cost amongst all available options, and an unused road/path has cost greater than or equal to the equilibrium cost. This is nothing but Wardrop's concept of user equilibrium and stated in complementarity form here (Equation 9).

$$0 \leq x \perp F(x) \geq 0 \qquad (9)$$

4. Proposed complementarity equilibrium computation framework

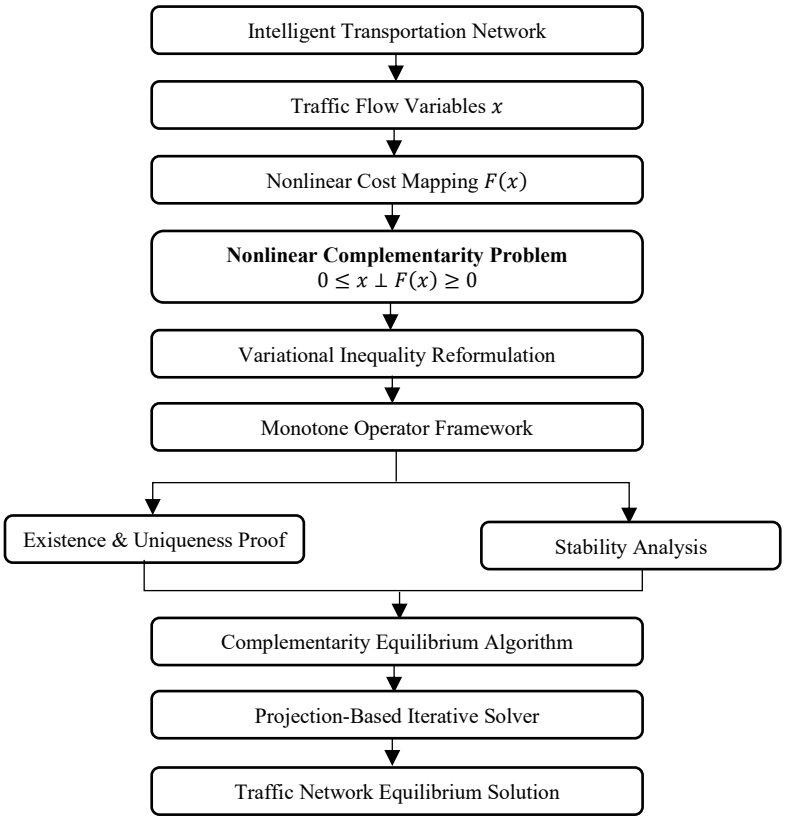


Figure 1: Conceptual framework of the complementarity-based equilibrium computation approach for intelligent transportation networks.

Figure 1 depicts the process of converting the traffic flow model variables into a nonlinear complementarity problem, a variational inequality model, and a monotone operator model through theory, solving for equilibrium and generating iterations through projections.

4.1. Complementarity residual transformation

The reformulation of the complementarity condition takes place by means of using the Fischer-Burmeister function φ that transforms any vector (a, b) into the zero vector if and only if the vector (a, b) satisfies the complementarity condition $a \geq 0, b \geq 0, ab = 0$ simultaneously. Using φ in a point-wise way for the vector $(x, F(x))$ allows to transform the original problem into a system of equations $\varphi\big(x, F(x)\big) = 0$ that does not have differentiability in the traditional sense but can be treated numerically (Equation 10).

$$\varphi(a, b) = \sqrt{a^2 + b^2} - a - b \qquad (10)$$

4.2. Merit function formulation

The merit function Ψ is defined as half the square of the Euclidean norm of the Fischer-Burmeister residual vector $\varphi\big(x, F(x)\big)$. The scalar function is smooth, nonnegative, and becomes zero only at the solution of the complementarity problem. Solving for the minimization of $\Psi(x)$ gives an equivalent smooth optimization problem to solve the original complementarity problem-based traffic equilibrium problem (Equation 11).

$$\Psi(x) = \frac{1}{2}\|\varphi\big(x, F(x)\big)\|^2 \qquad (11)$$

4.3. Projection-based equilibrium algorithm

The computation of equilibrium flow follows the process of an iterative gradient-type step against the cost operator F with respect to the current point, which is then projected to the feasible region using projection P. The choice of step size α with respect to the Lipschitz constant of F ensures that the resulting mapping is a contraction mapping under strong monotonicity and Lipschitz continuity of F, and forms the foundation of the convergence analysis in Section 5 (Equation 12).

$$x_{k+1} = P_K\big(x_k - \alpha F(x_k)\big) \qquad (12)$$

Algorithm 1. Projection-based complementarity equilibrium algorithm

Input: Initial flow $x_0 \in K$, step size $\alpha \in (0, 2\beta)$, tolerance ε_{tol}, maximum iterations $N_{\max}$.

Step 1: Set $k = 0$.

Step 2: Evaluate the traffic cost operator:

$$F(x_k) = T(x_k) + C$$

Step 3: Compute the trial point: $x_k - \alpha F(x_k)$.

Step 4: Project onto the feasible set:

$$x_{k+1} = P_K\big(x_k - \alpha F(x_k)\big)$$

Step 5: Evaluate the merit function $\Psi(x_{k+1})$ using the Fischer-Burmeister residual:

$$\Psi(x_{k+1}) = \frac{1}{2}\|\varphi\big(x_{k+1}, F(x_{k+1})\big)\|^2$$

Step 6: If $\Psi(x_{k+1}) \leq \varepsilon_{\text{tol}}$ or $k \geq N_{\max}$, stop and output x_{k+1} as the computed equilibrium.

Step 7: Otherwise, set $k = k + 1$ and return to Step 2.

The stopping criterion stops the execution of the algorithm when the merit function residual is less than a specified tolerance level ε_{tol} , implying that the complementarity condition has been met numerically, or when a predetermined maximum number of iterations $N_{\max}$has been met.

5. Convergence and stability analysis

5.1. Existence of equilibrium solution

Theorem 1

If the feasible set K is nonempty, compact, and convex, and the traffic cost operator F is continuous on K, then the traffic equilibrium complementarity problem admits at least one solution $x^* \in K$.

Proof:

The proof follows from Brouwer's fixed-point theorem applied to the projection operator $P_K\big(x - \alpha F(x)\big)$. The continuity of F and the compactness of K guarantee that a fixed point exists, which is shown to coincide exactly with a solution of the complementarity condition.

5.2. Uniqueness of equilibrium

Theorem 2

If, in addition to the conditions of Theorem 1, the operator F is strongly monotone on K with parameter $\mu > 0$, then the equilibrium solution x^* is unique.

Proof:

The proof considers two separate solutions and derives a contradiction straight away using the strong monotonicity inequality because the nonnegative lower bound on a quadratic expression forces the two potential solutions to be equal, showing that strong monotonicity is a sufficient condition for equilibrium uniqueness.

5.3. Convergence of proposed algorithm

Theorem 3. Suppose that F is μ-strongly monotone and L-Lipschitz, and $0 < \alpha < 2\mu/L^2$. Then the projection method exhibits linear convergence to the unique equilibrium point x^* according to the contraction property shown below with $\rho = \sqrt{(1 - 2\alpha\mu + \alpha^2 L^2)} < 1$. The proof uses the nonexpansiveness of the projection and strong monotonicity and Lipschitz continuity of F with respect to the update before projection (Equation 13).

$$\|x_{k+1} - x^*\| \leq \rho \|x_k - x^*\| \quad (13)$$

5.4. Perturbation and stability analysis

Real-world intelligent transportation systems suffer from measurement noise and fluctuations in demands, which in the current model are represented as an additive disturbance ε for the cost operator F. The equilibrium of the disturbed system x_ε lies within a bounded region around the original equilibrium x^*, assuming the strong monotonicity property holds with some parameter μ. The distance between x_ε and x^*scales linearly with the ratio of the size of the disturbance ε over μ (Equations 14 & 15).

$$F(x) + \varepsilon \quad (14)$$

$$\|x_\varepsilon - x^*\| \leq \frac{\varepsilon}{\mu} \quad (15)$$

6. Numerical experiments and computational evaluation

6.1. Experimental network setup

The experiments are performed on artificial and benchmark intelligent transportation systems with different sizes, starting from small-sized test networks and ending up with large networks of metropolitan size that have thousands of links and OD pairs. The traffic is generated according to peak hour distributions, and the costs of traveling through links are defined using the usual congestion cost functions that combine free flow time and volume/capacity ratio, thus making sure that the operator F is monotone and Lipschitz continuous.

Table 1 describes the structural parameters of the transportation networks used in the experiments. The transportation networks are classified according to an ascending scale, in terms of the number of nodes, links, OD pairs, and traffic intensity.

Table 1: Experimental network characteristics.

Network Instance	Nodes	Links	OD Pairs	Demand Level
Small Network	25	60	20	Low
Medium Network	120	350	150	Moderate
Large Network	500	1500	800	High
Metropolitan Network	2000	6500	3000	Very High

The growth in the size of the network allows to examine the effectiveness and computational performance of the proposed complementarity problem-based equilibrium solution technique. The chosen network models provide a variety of test conditions in order to verify the derived convergence and stability properties.

6.2. Comparison methods

The suggested projection-based complementarity algorithm is compared to three other algorithms: the Frank-Wolfe algorithm that makes use of convex-combination line search and separable cost functions, the Gradient Projection algorithm that uses projection but does not have the smoothed complementarity residual, and the Extragradient algorithm that is distinguished by greater stability under weaker conditions of monotonicity. All algorithms were run under identical conditions of network data, cost functions, and termination tolerances.

Table 2: Comparison of computational performance.

Method	Iterations	Equilibrium Gap	Residual Norm	Runtime (s)
Frank-Wolfe	420	8.7×10^{-5}	6.4×10^{-5}	18.6
Gradient Projection	260	4.2×10^{-5}	3.5×10^{-5}	12.4
Extragradient	190	2.8×10^{-5}	2.1×10^{-5}	10.2
Proposed Complementarity Method	95	7.6×10^{-7}	5.2×10^{-7}	5.8

The results of the computational performance of the equilibrium solving methods through iterations, gap of the equilibrium, complementarity residuals, and time are provided in table 2. The suggested complementarity method shows better convergence, lower residuals, and lower computational cost than traditional equilibrium solving methods.

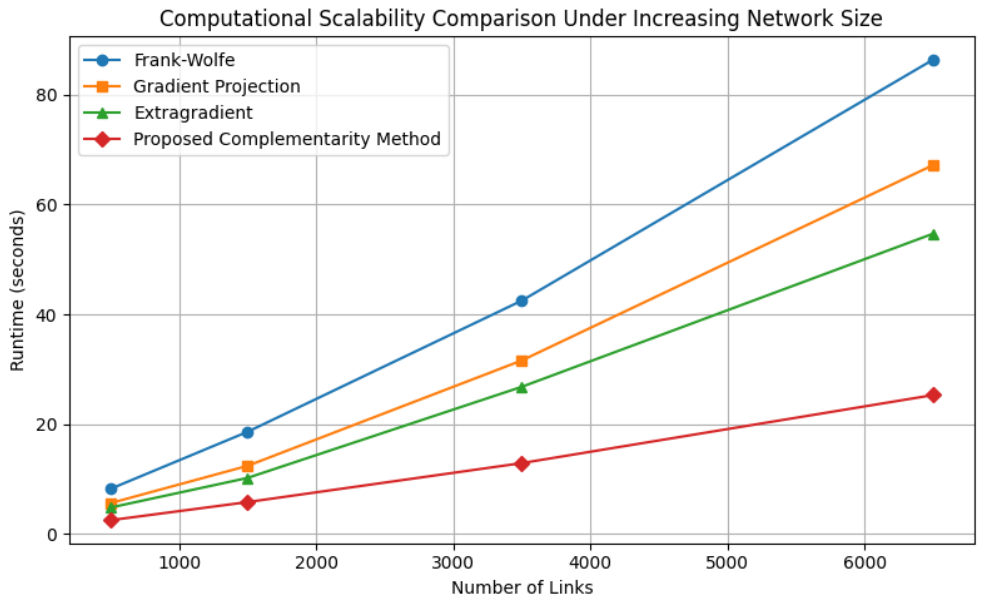


Figure 2: Computational scalability comparison under increasing transportation network size.

Figure 2 shows the runtime scalability of various approaches to equilibrium computation based on the growing number of network links. This comparative analysis proves that traditional algorithms like the Frank-Wolfe algorithm, the Gradient Projection algorithm, and the Extragradient algorithm are characterized by greater computational complexity with the increasing complexity of the network, while the approach to equilibrium computation remains computationally efficient.

7. Discussion

Complementarity problems and variational inequalities provide a more general mathematical framework for traffic equilibrium compared to the optimization approach due to the fact that it does not depend on symmetric or separable cost functions. Moreover, this approach allows analyzing non-symmetric network interrelationships without losing theoretical properties such as existence, uniqueness, and convergence of solutions, which are valid in the case of the operator F being strongly monotone, which means that there is a unique solution and a linear convergence rate, or is cocoercive.

The introduced projection method has good scalability properties since the computational complexity of the approach is defined by the sparsity of an operator and projection onto a feasible set. Convergence properties reduce the number of iterations for large-scale problems. This makes the approach suitable for metropolitan transport applications. However, under highly nonlinear congestion conditions, some adaptation, distributed computations, or decompositions may be needed for very large-scale networks.

8. Conclusion

The equilibrium computation framework proposed in this paper utilizes traffic flow management as a nonlinear complementarity problem with variational inequality reformulation. Specifically, the Fischer-Burmeister residual transformation method, merit function minimization, and projection iterations are incorporated into the framework. The theoretical analysis revealed that an equilibrium exists for continuous problems, a unique equilibrium is ensured for strongly monotone problems, and linear convergence of the proposed approach is possible due to monotonicity properties. The numerical experiments have shown that the proposed approach yields smaller equilibrium gaps, lower complementarity residuals, and higher efficiency than Frank-Wolfe, Gradient Projection, and Extragradient approaches. This evidence confirms that the proposed framework is a generalized mathematical tool that is not limited to optimization-based traffic assignment.

Future research may extend the proposed framework into the dynamic and stochastic transportation environment in which uncertain demand, traffic variations, and adaptive mobility behavior are modeled as part of the equilibrium problem. It is also possible to investigate distributed complementarity algorithms that would increase the scalability of the framework and allow solving extremely large-scale problems of traffic equilibrium in transportation networks that contain multiple decision agents. Other extensions might include learning-assisted estimation of the operators, multi-modal transportation problems, and physics-informed hybrid traffic equilibrium problems.

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