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Optimizing Healthcare Operations: A Pentapartitioned Neutrosophic Vague Set Approach

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Abstract
By fusing the concepts of Neutrosophic Vague set and Pentapartitioned Neutrosophic set, this paper aims to define the term Pentapartitioned Neutrosophic Vague set. Also, we define definitions, provide examples and properties of the Pentapartitioned Neutrosophic Vague Set. We have developed a novel idea called "Pentapartitioned Neutrosophic vague topological spaces" and spoken about some of its characteristics. Also we study the application in medical diagnosis using normalized Euclidean distance method.
Keywords: Neutrosophic set, Neutrosophic vague set, Pentapartitioned Neutrosophic set, Normalized Euclidean distance.

1.Introduction

The idea of a fuzzy set was initially suggested in 1965 by Lofti A. Zadeh [19]. The key idea behind this approach is that it establishes the uncertainty of a set via a membership function that provides values to the components of the universal set that fall between [0,1]. These metrics are referred to as membership degrees. In 1986 Atanassov [2] introduced the Intuitionistic Fuzzy Set, which expands on the Fuzzy Set concept by additionally taking into account the non-membership degree. In this case, the cardinality of a finite member is indicated by the letters.
By proposing the idea of a Neutrosophic set, Smarandache expanded the notion of an intuitionistic fuzzy set (NS). Smarandache originally discussed Neutrosophy in 1995, and in 2005 [8] he defined the Neutrosophic set theory, one of the most significant techniques for dealing with issues involving erroneous, ambiguous, and inconsistent data. Many scholars go on to employ NS in both their theoretical and applied research after then.
The term "Neutrosophic vague set," which combines the terms "Neutrosophic set" and "vague set," was first introduced by Shawkat Alkhazaleh [1] in 2015. Neutrosophic vague theory is a useful method for analysing ambiguous, conflicting, and incomplete data. Indeterminacy was divided into three categories by Smarandache [9]: unknown, contradiction, and ignorance. He also developed Five Symbol Valued Neutrosophic Logic (FSVNL).
In this work, we provide definitions and some instances for the Pentapartitioned Neutrosophic Vague Set's operations and characteristics. We create a new notion called "Pentapartitioned Neutrosophic vague topological spaces," describe some of its aspects, and employ Pentapartitioned Neutrosophic ambiguous set and topological spaces in our work.

2.Preliminaries

Definition 2.1: [1]
Let X be a nonempty set. A ***Fuzzy set*** *A in X is given by*

$$A = \{(x, \mu_A(x)) / x \in X\}$$

Where $\mu_A(x) : X \to [0,1]$ is the *membership function* of the Fuzzy Set A.
$(i.e)$, $\mu_A(x) \in [0,1]$ *is the membership of* $x \in X$ *in* A.

Definition 2.2 :[2]
An ***Intuitionistic Fuzzy Set (IFS)*** , A in X *is given by* $A = \{(x, \mu_A(x), \vartheta_A(x)) / x \in X\}$ *where* $\mu_A : X \to [0,1]$ *and* $\vartheta_A : X \to [0,1]$ with the condition $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1, \forall x \in X$. Here $\mu_A(x)$ and $\vartheta_A(x) \in [0,1]$.

Definition 2.3:[4]
Let U be a universe. A ***Neutrosophic set*** *A on X is*

$$A = \{(x, T_A(x), I_A(x), F_A(x)) / x \in X\}$$

where $T, I, F : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. Here $T_A(x)$ *is the degree of membership*, $I_A(x)$ *is the degree of indeterminacy and* $F_A(x)$ *is the degree of non − membership.*

Definition 2.4: [7]

A ***Neutrosophic vague set*** *(NVS) on X written as*

$A = \{\langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle; x \in X\}$, *whose truth membership, indeterminacy membership and false membership functions are defined as:*

$\hat{T}_A(x) = [T^-, T^+]$, $\hat{I}_A(x)[I^-, I^+]$, $\hat{F}_A(x) = [F^-, F^+]$,

Where, (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ and (3)

$0 \le T^- + I^- + F^- \le 2^+$ *when X is continuous*, a *NVS* is

$$A = \int \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle / x, x \in X$$

When X is discrete, a NVS is

$$A = \sum_{i=1}^{n} \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle / x_i,\ x_i \in X$$

Definition 2.5: [8]

A **Pentapartitioned Neutrosophic set** A on X is defined as

$A = \{\langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x)\rangle / x \in X\}$

$T, C, G, U, F : X \to [0,1]$, $0 \le T_A(x) + C_A(x) + G_A(x) + U_A(x) + F_A(x) \le 5$. Here $T_A(x)$ *is the truth membership*, $C_A(x)$ *is contradiction membership*, $G_A(x)$ *is ignorance* , $U_A(x)$ *is unknown and* $F_A(x)$ *is the false membership.*

Definition 2.6: [10]

The normalized Euclidean distance for neutrosophic *sets A and B in* $X = \{x_1, x_2, \dots \dots \dots x_n\}$ is

$$\left(\frac{1}{n}\sum_{i=1}^{n}\left(T^+_{A_{NV}}(x_i) - T^+_{B_{NV}}(x_i)\right|\right)^2 + \left(T^-_{A_{NV}}(x_i) - T^-_{B_{NV}}(x_i)\right|\right)^2 + \left(I^+_{A_{NV}}(x_i) - I^+_{B_{NV}}(x_i)\right|\right)^2 + \left(I^-_{A_{NV}}(x_i) - I^-_{B_{NV}}(x_i)\right|\right)^2$$
$$+ \left(F^+_{A_{NV}}(x_i) - F^+_{B_{NV}}(x_i)\right|\right)^2 + \left(F^-_{A_{NV}}(x_i) - F^-_{B_{NV}}(x_i)\right|\right)^2\Bigg)^{\frac{1}{2}}$$

3. Pentapartitioned Neutrosophic Vague Set

Definition 3.1:

A Pentapartitioned Neutrosophic Vague Set $A_{\widetilde{PNV}}$ **($\widetilde{PNVS}$)** on the universe is,

$$A_{\widetilde{PNV}} = \left\{\langle x; \hat{T}_{A_{\widetilde{PNV}}}(x); \hat{C}_{A_{\widetilde{PNV}}}(x); \hat{G}_{A_{\widetilde{PNV}}}(x); \hat{U}_{A_{PNV}}(x); \hat{F}_{A_{\widetilde{PNV}}}(x)\rangle; x \in X\right\}$$

where

$\hat{T}_{A_{\widetilde{PNV}}}(x) = [T^-, T^+]$, $\hat{C}_{A_{\widetilde{PNV}}}(x) = [C^-, C^+]$, $\hat{G}_{A_{\widetilde{PNV}}}(x) = [G^-, G^+]$,

$\hat{U}_{A_{\widetilde{PNV}}}(x) = [U^-, U^+]$, $\hat{F}_{A_{\widetilde{PNV}}}(x) = [F^-, F^+]$,

Where (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $C^+ = 1 - U^-$ (4) $U^+ = 1 - C^-$

(5) $0 \le T^- + C^- + G^- + U^- + F^- \le 3^+$

Example 3.2:

Let $U = \{x, y\}$ be a set. We define the $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$:

$$A_{\widetilde{PNV}} = \left\{\frac{x}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}}, \frac{y}{\{[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]\}}\right\}$$

Definition 3.3:

Let $A_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}}$ be two of the universe U. If $\forall x \in U$,

$\hat{T}_{A_{\widetilde{PNV}}}(x) \le \hat{T}_{B_{\widetilde{PNV}}}(x)$; $\hat{C}_{A_{\widetilde{PNV}}}(x) \le \hat{C}_{B_{\widetilde{PNV}}}(x)$; $\hat{G}_{A_{\widetilde{PNV}}}(x) \ge \hat{G}_{B_{\widetilde{PNV}}}(x)$; $\hat{U}_{A_{\widetilde{PNV}}}(x) \ge \hat{U}_{B_{\widetilde{PNV}}}(x)$; $\hat{F}_{A_{\widetilde{PNV}}}(x) \ge \hat{F}_{B_{\widetilde{PNV}}}(x)$

then the $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ *is included by* B_{PNV}, *denoted by* $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}}$.

Definition 3.4:

The complement of $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ is $A^c_{\widetilde{PNV}}$.

$\hat{T}^c_{A_{\widetilde{PNV}}}(x) = [1 - T^+, 1 - T^-]$, $\hat{C}^c_{A_{\widetilde{PNV}}}(x) = [1 - C^+, 1 - C^-]$, $\hat{G}^c_{A_{\widetilde{PNV}}}(x) = [1 - G^+, 1 - G^-]$

$\hat{U}^c_{A_{\widetilde{PNV}}}(x) = [1 - U^+, 1 - U^-]$, $\hat{F}^c_{A_{\widetilde{PNV}}}(x) = [1 - F^+, 1 - F^-]$

Example 3.5:

Let $U = \{x, y\}$ *be a set. We define the* $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ as :

$$A_{\widetilde{PNV}} = \left\{\frac{x}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}}, \frac{y}{\{[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]\}}\right\}$$

Then the complement of $A_{\widetilde{PNV}}$ is

$$A^c_{\widetilde{PNV}} = \left\{\frac{x}{\{[0.4,0.7],[0.5,0.6],[0.6,0.8],[0.4,0.5],[0.3,0.6]\}}, \frac{y}{\{[0.5,0.7],[0.4,0.8],[0.5,0.9],[0.2,0.6],[0.3,0.5]\}}\right\}$$

Definition 3.6:

Let $A_{\widetilde{PNV}}$ *be a* $\widetilde{PNVS}$ *on U where* $\forall x \in U$, $\hat{T}_{A_{\widetilde{PNV}}}(x) = [1,1]$, $\hat{C}_{A_{\widetilde{PNV}}}(x) = [1,1]$, $\hat{G}_{A_{\widetilde{PNV}}}(x) = [0,0]$,

$\hat{U}_{A_{\widetilde{PNV}}}(x) = [0,0]$, $\hat{F}_{A_{\widetilde{PNV}}}(x) = [0,0]$. *Then* $A_{\widetilde{PNV}}$ *is called unit* $\widetilde{PNVS}$ $(1_{\widetilde{PNV}})$.

Definition 3.7:

Let $A_{\widetilde{PNV}}$ *be a* $\widetilde{PNVS}$ *on* U *where* $\forall x \in U$, $\hat{T}_{A_{\widetilde{PNV}}}(x) = [0,0]$, $\hat{C}_{A_{\widetilde{PNV}}}(x) = [0,0]$, $\hat{G}_{A_{\widetilde{PNV}}}(x) = [1,1]$, $\hat{U}_{A_{\widetilde{PNV}}}(x) = [1,1]$, $\hat{F}_{A_{\widetilde{PNV}}}(x) = [1,1]$. *Then* $A_{\widetilde{PNV}}$ *is called zero* $\widetilde{PNVS}$ $(0_{\widetilde{PNV}})$.

Definition 3.8:

Let $A_{\widetilde{PNV}}$ *and* $B_{\widetilde{PNV}}$ *be two* $\widetilde{PNVS}$ *of the universe* U. *If* $\forall x \in U$,

$\hat{T}_{A_{\widetilde{PNV}}}(x) = \hat{T}_{B_{\widetilde{PNV}}}(x)$;

$\hat{C}_{A_{\widetilde{PNV}}}(x) = \hat{C}_{B_{\widetilde{PNV}}}(x)$; $\hat{G}_{A_{\widetilde{PNV}}}(x) = \hat{G}_{B_{\widetilde{PNV}}}(x)$; $\hat{U}_{A_{\widetilde{PNV}}}(x) = \hat{U}_{B_{\widetilde{PNV}}}(x)$; $\hat{F}_{A_{\widetilde{PNV}}}(x) = \hat{F}_{B_{\widetilde{PNV}}}(x)$ then the $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}}$ are equal.

Definition 3.9:

The union of two $\widetilde{PNVS}$*s* $A_{\widetilde{PNV}}$ *and* B_{PNV} *is a* $\widetilde{PNVS}$ $K_{\widetilde{PNV}}$

$K_{\widetilde{PNV}} = A_{\widetilde{PNV}} \cup B_{\widetilde{PNV}}$ *where*

$$\hat{T}_{K_{\widetilde{PNV}}}(x) = \left[max\left(\hat{T}^{-}_{A_{\widetilde{PNV}}}, \hat{T}^{-}_{B_{\widetilde{PNV}}}\right), max\left(\hat{T}^{+}_{A_{\widetilde{PNV}}}, \hat{T}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{C}_{K_{\widetilde{PNV}}}(x) = \left[max\left(\hat{C}^{-}_{A_{\widetilde{PNV}}}, \hat{C}^{-}_{B_{\widetilde{PNV}}}\right), max\left(\hat{C}^{+}_{A_{\widetilde{PNV}}}, \hat{C}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{G}_{K_{\widetilde{PNV}}}(x) = \left[min\left(\hat{G}^{-}_{A_{\widetilde{PNV}}}, \hat{G}^{-}_{B_{\widetilde{PNV}}}\right), min\left(\hat{G}^{+}_{A_{\widetilde{PNV}}}, \hat{G}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{U}_{K_{\widetilde{PNV}}}(x) = \left[min\left(\hat{U}^{-}_{A_{\widetilde{PNV}}}, \hat{U}^{-}_{B_{\widetilde{PNV}}}\right), min\left(\hat{U}^{+}_{A_{\widetilde{PNV}}}, U^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{F}_{K_{\widetilde{PNV}}}(x) = \left[min\left(\hat{F}^{-}_{A_{\widetilde{PNV}}}, \hat{F}^{-}_{B_{\widetilde{PNV}}}\right), min\left(\hat{F}^{+}_{A_{\widetilde{PNV}}}, F^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

Example 3.10:

Let $U = \{u_1, u_2\}$ *and let* $A_{\widetilde{PNV}}$ *and* $B_{\widetilde{PNV}}$ be $\widetilde{PNVS}$ *defined as follows:*

$$A_{\widetilde{PNV}} = \left\{ \frac{u_1}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}}, \frac{u_2}{\{[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]\}} \right\}$$

$$B_{\widetilde{PNV}} = \left\{ \frac{u_1}{\{[0.2,0.6],[0.3,0.5],[0.6,0.8],[0.5,0.7],[0.4,0.8]\}}, \frac{u_2}{\{[0.35,0.7],[0.55,0.7],[0.15,0.6],[0.3,0.45],[0.3,0.65]\}} \right\}$$

Then we have $K_{\widetilde{PNV}} = A_{\widetilde{PNV}} \cup B_{\widetilde{PNV}}$

$$= \left\{ \frac{u_1}{\{[max(0.3,0.2), max(0.6,0.6)], [max(0.4,0.3), max(0.5,0.5)], [min(0.2,0.6), min(0.4,0.8)], [min(0.5,0.5), min(0.6,0.7), min(0.4,0.4), min(0.7,0.8)]\}}, \frac{u_2}{[[max(0.3,0.35), max(0.5,0.7)], [max(0.2,0.55), max(0.6,0.7)], [min(0.1,0.15), min(0.5,0.6)], [min(0.4,0.3), min(0.8,0.45), min(0.5,0.3), min(0.7,0.65)]]} \right\}$$

$$K_{\widetilde{PNV}} = \left\{ \frac{u_1}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}}, \frac{u_2}{\{[0.35,0.7],[0.55,0.7],[0.1,0.5],[0.3,0.45],[0.3,0.65]\}} \right\}$$

Definition 3.11:

The intersection of two $\widetilde{PNVS}$*s* $A_{\widetilde{PNV}}$ *and* $B_{\widetilde{PNV}}$ is a $\widetilde{PNVS}$ $H_{\widetilde{PNV}}$,

$H_{\widetilde{PNV}} = A_{\widetilde{PNV}} \cap B_{\widetilde{PNV}}$ *,where*

$$\hat{T}_{H_{\widetilde{PNV}}}(x) = \left[min\left(\hat{T}^{-}_{A_{\widetilde{PNV}}}, \hat{T}^{-}_{B_{\widetilde{PNV}}}\right), min\left(\hat{T}^{+}_{A_{\widetilde{PNV}}}, \hat{T}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{C}_{H_{\widetilde{PNV}}}(x) = \left[min\left(\hat{C}^{-}_{A_{\widetilde{PNV}}}, \hat{C}^{-}_{B_{\widetilde{PNV}}}\right), min\left(\hat{C}^{+}_{A_{\widetilde{PNV}}}, \hat{C}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{G}_{H_{\widetilde{PNV}}}(x) = \left[max\left(\hat{G}^{-}_{A_{\widetilde{PNV}}}, \hat{G}^{-}_{B_{\widetilde{PNV}}}\right), max\left(\hat{G}^{+}_{A_{\widetilde{PNV}}}, \hat{G}^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{U}_{H_{\widetilde{PNV}}}(x) = \left[max\left(\hat{U}^{-}_{A_{\widetilde{PNV}}}, \hat{U}^{-}_{B_{\widetilde{PNV}}}\right), max\left(\hat{U}^{+}_{A_{\widetilde{PNV}}}, U^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

$$\hat{F}_{H_{\widetilde{PNV}}}(x) = \left[max\left(\hat{F}^{-}_{A_{\widetilde{PNV}}}, \hat{F}^{-}_{B_{\widetilde{PNV}}}\right), max\left(\hat{F}^{+}_{A_{\widetilde{PNV}}}, F^{+}_{B_{\widetilde{PNV}}}\right)\right]$$

Example 3.12:

Let $U = \{u_1, u_2\}$ *and let* $A_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}}$ be $\widetilde{PNVS}$*s defined as follows:*

$$A_{\widetilde{PNV}} = \left\{ \frac{u_1}{[[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]]}, \frac{u_2}{[[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]]} \right\}$$

$$B_{\widetilde{PNV}} = \left\{ \frac{u_1}{[[0.2,0.6],[0.3,0.5],[0.6,0.8],[0.5,0.7],[0.4,0.8]]}, \frac{u_2}{[[0.35,0.7],[0.55,0.7],[0.15,0.6],[0.3,0.45],[0.3,0.65]]} \right\}$$

Then we have $H_{\widetilde{PNV}} = A_{\widetilde{PNV}} \cap B_{\widetilde{PNV}}$

$$= \left\{ \begin{matrix} \dfrac{u_1}{\left\{ \begin{matrix} [\min(0.3,0.2),\min(0.6,0.6)],[\min(0.4,0.3),\min(0.5,0.5)],[\max(0.2,0.6),\max(0.4,0.8)], \\ [\max(0.5,0.5),\max(0.6,0.7),\max(0.4,0.4),\max(0.7,0.8)] \end{matrix} \right\}}, \\ \dfrac{u_2}{\left\{ \begin{matrix} [\min(0.3,0.35),\min(0.5,0.7)],[\min(0.2,0.55),\min(0.6,0.7)],[\max(0.1,0.15),\max(0.5,0.6)], \\ [\max(0.4,0.3),\max(0.8,0.45),\max(0.5,0.3),\max(0.7,0.65)] \end{matrix} \right\}} \end{matrix} \right\}$$

$$H_{\widetilde{PNV}} = \left\{ \frac{u_1}{\{[0.2,0.6],[0.3,0.5],[0.6,0.8],[0.5,0.7],[0.4,0.8]\}}, \frac{u_2}{\{[0.3,0.5],[0.2,0.6],[0.15,0.6],[0.4,0.8],[0.5,0.7]\}} \right\}$$

Definition 3.13:

Let $\left\{A_{i_{\widetilde{PNV}}} : i \in J\right\}$ *be an arbitrary family of* $\widetilde{PNVSs}$.

$$\bigcup A_{i_{\widetilde{PNV}}} = \left\{ \begin{matrix} x; \left(\max_{i\in J}\left(\tilde{T}^-_{A_{i_{\widetilde{PNV}}}}\right), \max_{i\in J}\left(\tilde{T}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \left(\max_{i\in J}\left(\tilde{C}^-_{A_{i_{\widetilde{PNV}}}}\right), \max_{i\in J}\left(\tilde{C}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \\ \left(\min_{i\in J}\left(\tilde{G}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{G}^+_{A_{i_{\widetilde{PNV}}}}\right) \right) \\ \left(\min_{i\in J}\left(\tilde{U}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{U}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \left(\min_{i\in J}\left(\tilde{F}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{F}^+_{A_{i_{\widetilde{PNV}}}}\right) \right) \end{matrix} : x \in X \right\}$$

$$\bigcap A_{i_{\widetilde{PNV}}} = \left\{ \begin{matrix} x; \left(\max_{i\in J}\left(\tilde{T}^-_{A_{i_{\widetilde{PNV}}}}\right), \max_{i\in J}\left(\tilde{T}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \left(\max_{i\in J}\left(\tilde{C}^-_{A_{i_{\widetilde{PNV}}}}\right), \max_{i\in J}\left(\tilde{C}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \\ \left(\min_{i\in J}\left(\tilde{G}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{G}^+_{A_{i_{\widetilde{PNV}}}}\right) \right) \\ \left(\min_{i\in J}\left(\tilde{U}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{U}^+_{A_{i_{\widetilde{PNV}}}}\right) \right), \left(\min_{i\in J}\left(\tilde{F}^-_{A_{i_{\widetilde{PNV}}}}\right), \min_{i\in J}\left(\tilde{F}^+_{A_{i_{\widetilde{PNV}}}}\right) \right) \end{matrix} : x \in X \right\}$$

Theorem 3.14:

Let $A_{\widetilde{PNV}}, B_{\widetilde{PNV}}, C_{\widetilde{PNV}}, D_{\widetilde{PNV}}, E_{\widetilde{PNV}}$ be $\widetilde{PNVSs}$. Then

(i) $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}}$ and $C_{\widetilde{PNV}} \subseteq D_{\widetilde{PNV}} \Rightarrow A_{\widetilde{PNV}} \cup C_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \cup D_{\widetilde{PNV}}$ and $A_{\widetilde{PNV}} \cap C_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \cap D_{\widetilde{PNV}}$

(ii) $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}}$ and $A_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}} \Rightarrow A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \cap C_{\widetilde{PNV}}$

(iii) $A_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}} \Rightarrow A_{\widetilde{PNV}} \cup B_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}}$

(iv) $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}}$ and $B_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}} \Rightarrow A_{\widetilde{PNV}} \subseteq C_{\widetilde{PNV}}$

(v) $\overline{(A_{\widetilde{PNV}} \cup B_{\widetilde{PNV}})} = \overline{(A_{\widetilde{PNV}})} \cap \overline{(B_{\widetilde{PNV}})}$

(vi) $\overline{(A_{\widetilde{PNV}} \cap B_{\widetilde{PNV}})} = \overline{(A_{\widetilde{PNV}})} \cup \overline{(B_{\widetilde{PNV}})}$

(vii) $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \Rightarrow \overline{B_{\widetilde{PNV}}} \subseteq \overline{A_{\widetilde{PNV}}}$

(viii) $\overline{(\overline{A_{\widetilde{PNV}}})} = A_{\widetilde{PNV}}$

(ix) $\overline{1}_{\widetilde{PNV}} = 0_{\widetilde{PNV}}$

(x) $\overline{0}_{\widetilde{PNV}} = 1_{\widetilde{PNV}}$

Corollary 3.15:

Let $A_{\widetilde{PNV}}, B_{\widetilde{PNV}}, C_{\widetilde{PNV}}, D_{\widetilde{PNV}}$, and $\left\{A_{i_{\widetilde{PNV}}} : i \in J\right\}$ be $\widetilde{PNVSs}$. Then

(i) $A_{i_{\widetilde{PNV}}} \subseteq B_{\widetilde{PNV}}\ \forall\, i \in J \Rightarrow \cup A_{i_{\widetilde{PNV}}} \subseteq B_{\widetilde{PNV}}$

(ii) $B_{\widetilde{PNV}} \subseteq A_{i_{\widetilde{PNV}}} \forall\, i \in J \Rightarrow B_{\widetilde{PNV}} \subseteq \cap A_{i_{\widetilde{PNV}}}$

(iii) $\overline{\cup A_{i_{\widetilde{PNV}}}} = \cap \overline{A_{i_{\widetilde{PNV}}}}$

(iv) $\overline{\cap A_{i_{\widetilde{PNV}}}} = \cup A_{i_{\widetilde{PNV}}}$

4.Pentapartitioned Neutrosophic Vague Topological Spaces

Definition 4.1:

A Pentapartitioned *Neutrosophic vague topology* ($\widetilde{PNVT}$) on $X_{\widetilde{PNV}}$ is $\tau_{\widetilde{PNV}}$ of ($\widetilde{PNVS}$) in $X_{\widetilde{PNV}}$ *satisfying the following axioms*:

(i) $0_{\widetilde{PNV}}, 1_{\widetilde{PNV}} \in \tau_{\widetilde{PNV}}$

(ii) $N_1 \cap N_2 \in \tau_{\widetilde{PNV}}$, *for any* $N_1, N_2 \in \tau_{\widetilde{PNV}}$

(iii) $\cup N_i \in \tau_{\widetilde{PNV}}, \forall\{N_i : i \in J\} \subseteq \tau_{\widetilde{PNV}}$

$(X_{\widetilde{PNV}}, \tau_{\widetilde{PNV}})$ *is called Pentapartitioned Neutrosophic vague topological space* ($\widetilde{PNVTS}$).

Example 4.2:

Let $X_{\widetilde{PNV}} = \{e, f\}$ *and*

$$A_{\widetilde{PNV}} = \left\{ \begin{matrix} \dfrac{e}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}} \\ \dfrac{f}{\{[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]\}} \end{matrix} \right\}$$

$$B_{\widetilde{PNV}} = \left\{ \frac{e}{\{[0.2,0.6],[0.3,0.5],[0.6,0.8],[0.5,0.7],[0.4,0.8]\}}, \frac{f}{\{[0.35,0.7],[0.55,0.7],[0.15,0.6],[0.3,0.45],[0.3,0.65]\}} \right\}$$

$$C_{\widetilde{PNV}=} \left\{ \frac{e}{\{[0.3,0.5],[0.3,0.7],[0.2,0.5],[0.3,0.7],[0.5,0.7]\}}, \frac{f}{\{[0.2,0.75],[0.7,0.8],[0.5,0.65],[0.2,0.3],[0.25,0.8]\}} \right\}$$

Then the family of $\tau_{\widetilde{PNV}} = \{0_{\widetilde{PNV}}, A_{\widetilde{PNV}}, B_{\widetilde{PNV}}, C_{\widetilde{PNV}}, 1_{\widetilde{PNV}}\}$ of $\widetilde{PNVSs}$ in $X_{\widetilde{PNV}}$ is $\widetilde{PNVT}$ on $X_{\widetilde{PNV}}$.

Definition 4.3:

Let $(X_{\widetilde{PNV}}, \tau_{\widetilde{PNV}})$ be $\widetilde{PNVTS}$ and

$A_{\widetilde{PNV}} = \{\langle x, [\hat{T}_A(x); \hat{C}_A(x); \hat{G}_A(x); \hat{U}_A(x); \hat{F}_A(x)]\rangle\}$

$\widetilde{PNVS}$ in $X_{\widetilde{PNV}}$. *Then the Pentapartitioned Neutrosophic Vague interior and closure are defined by*

(i) $\widetilde{PNV}\ int(A_{\widetilde{PNV}}) = \cup\{N_{\widetilde{PNV}} / N_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVOS}\ in\ X_{\widetilde{PNV}}\ and\ N_{\widetilde{PNV}} \subseteq A_{\widetilde{PNV}}\}$

(ii) $\widetilde{PNV}\ cl(A_{\widetilde{PNV}}) = \cap\{M_{\widetilde{PNV}} / M_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVCS}\ in\ X_{\widetilde{PNV}}\ and\ A_{\widetilde{PNV}} \subseteq M_{\widetilde{PNV}}\}$

Also for any $\widetilde{PNVS}$ $A_{\widetilde{PNV}}$ in $(X_{\widetilde{PNV}}, \tau_{\widetilde{PNV}})$ we have $\widetilde{PNV}\ cl(A^c_{\widetilde{PNV}}) = \left(\widetilde{PNV}\ int(A_{\widetilde{PNV}})\right)^c$ and $\widetilde{PNV}\ int(A^c_{\widetilde{PNV}}) = \left(\widetilde{PNV}\ cl(A_{\widetilde{PNV}})\right)^c$.

It can also be shown that $\widetilde{PNV}\ cl(A_{\widetilde{PNV}})$ is $\widetilde{PNVCS}$ and $\widetilde{PNV}\ int(A_{\widetilde{PNV}})$ is $\widetilde{PNVOS}$ in $X_{\widetilde{PNV}}$.

a) $A_{\widetilde{PNV}}$ is $\widetilde{PNVCS}$ in $X_{\widetilde{PNV}}$ if and only if $\widetilde{PNV}\ cl(A_{\widetilde{PNV}}) = A_{\widetilde{PNV}}$.

b) $A_{\widetilde{PNV}}$ is $\widetilde{PNVOS}$ in $X_{\widetilde{PNV}}$ if and only if $\widetilde{PNV}\ int(A_{\widetilde{PNV}}) = A_{\widetilde{PNV}}$.

Example 4.4:

Let $X_{\widetilde{PNV}} = \{e, f\}$ and let $\tau_{\widetilde{PNV}} = \{0_{\widetilde{PNV}}, N_1, N_2, 1_{\widetilde{PNV}}\}$ be $PNVT$ on X, where

$$N_1 = \left\{ \frac{e}{\{[0.1,0.5],[0.3,0.3],[0.2,0.4],[0.7,0.7],[0.5,0.9]\}}, \frac{f}{\{[0.4,0.55],[0.2,0.6],[0.3,0.5],[0.3,0.4],[0.45,0.6]\}} \right\}$$

$$N_2 = \left\{ \frac{e}{\{[0.3,0.6],[0.4,0.5],[0.2,0.4],[0.5,0.6],[0.4,0.7]\}}, \frac{f}{\{[0.3,0.5],[0.2,0.6],[0.1,0.5],[0.4,0.8],[0.5,0.7]\}} \right\}$$

If $A_{\widetilde{PNV}} = \left\{ \frac{e}{\{[0.2,0.6],[0.4,0.7],[0.5,0.8],[0.3,0.6],[0.4,0.8]\}}, \frac{f}{\{[0.35,0.5],[0.15,0.65],[0.45,0.75],[0.35,0.85],[0.5,0.65]\}} \right\}$,

then

$$\widetilde{PNV}\ int(A_{\widetilde{PNV}}) = N_1 = \left\{ \frac{e}{\{[0.1,0.5],[0.3,0.3],[0.2,0.4],[0.7,0.7],[0.5,0.9]\}}, \frac{f}{\{[0.4,0.55],[0.2,0.6],[0.3,0.5],[0.3,0.4],[0.45,0.6]\}} \right\}$$

$$\widetilde{PNV}\ cl(A_{\widetilde{PNV}}) = N_1^c = \left\{ \frac{e}{\{[0.5,0.9],[0.7,0.7],[0.6,0.8],[0.3,0.3],[0.1,0.5]\}}, \frac{f}{\{[0.45,0.6],[0.4,0.8],[0.5,0.7],[0.6,0.7],[0.4,0.55]\}} \right\}$$

Theorem 4.5:

Let $A_{\widetilde{PNV}}$ be any $\widetilde{PNVS}$ in $X_{\widetilde{PNV}}$, then

(i) $\widetilde{PNV}\ int(1_{\widetilde{PNV}} - A_{\widetilde{PNV}}) = 1_{\widetilde{PNV}} - \left(\widetilde{PNV}\ cl(A_{\widetilde{PNV}})\right)$ and

(ii) $\widetilde{PNV}\ cl(1_{\widetilde{PNV}} - A_{\widetilde{PNV}}) = 1_{\widetilde{PNV}} - \left(\widetilde{PNV}\ int(A_{\widetilde{PNV}})\right)$

Proof: (i) By Definition $\widetilde{PNV}\ cl(A_{\widetilde{PNV}}) = \cap\{M_{\widetilde{PNV}} / M_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVCS}\ in\ X_{\widetilde{PNV}}\ and\ A_{\widetilde{PNV}} \subseteq M_{\widetilde{PNV}}\}$

Then $1_{\widetilde{PNV}} - \left(\widetilde{PNV}\ cl(A_{\widetilde{PNV}})\right) = 1_{\widetilde{PNV}} - \cap\{M_{\widetilde{PNV}} / M_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVCS}\ in\ X_{\widetilde{PNV}}\ and\ A_{\widetilde{PNV}} \subseteq M_{\widetilde{PNV}}\}$

$= \cup\{1_{\widetilde{PNV}} - M_{\widetilde{PNV}} / M_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVCS}\ in\ X_{\widetilde{PNV}}\ and\ A_{\widetilde{PNV}} \subseteq M_{\widetilde{PNV}}\}$

$= \cup\{N_{\widetilde{PNV}} / N_{\widetilde{PNV}}\ is\ a\ \widetilde{PNVOS}\ in\ X_{\widetilde{PNV}}\ and\ N_{\widetilde{PNV}} \subseteq 1_{\widetilde{PNV}} - A_{\widetilde{PNV}}\}$

$= \widetilde{PNV}\ int(1_{\widetilde{PNV}} - A_{\widetilde{PNV}})$

(iii) The proof is similar to (i).

Theorem 4.6:

Let $(X_{\widetilde{PNV}}, \tau_{\widetilde{PNV}})$ be $\widetilde{PNVTS}$ and $A_{\widetilde{PNV}}$, $B_{\widetilde{PNV}}$ be $\widetilde{PNVSs}$ in $X_{\widetilde{PNV}}$. Then the following properties hold:

a) $\widetilde{PNV}\ int(A_{\widetilde{PNV}}) \subseteq A_{\widetilde{PNV}}$

$a_1)$ $A_{\widetilde{PNV}} \subseteq \widetilde{PNV}\ cl(A_{\widetilde{PNV}})$

b) $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \Rightarrow \widetilde{PNV}\ int(A_{\widetilde{PNV}}) \subseteq \widetilde{PNV}\ int(B_{\widetilde{PNV}})$

$b_1)$ $A_{\widetilde{PNV}} \subseteq B_{\widetilde{PNV}} \Rightarrow \widetilde{PNV}\ cl(A_{\widetilde{PNV}}) \subseteq \widetilde{PNV}\ cl(B_{\widetilde{PNV}})$

c) $\widetilde{PNV}\ int\left(\widetilde{PNV}\ int\ (A_{\widetilde{PNV}})\right) = \widetilde{PNV}\ int\ (A_{\widetilde{PNV}})$

$c_1)$ $\widetilde{PNV}\ cl\left(\widetilde{PNV}\ cl\ (A_{\widetilde{PNV}})\right) = \widetilde{PNV}\ cl\ (A_{\widetilde{PNV}})$

d) $\widetilde{PNV}\ int(A_{\widetilde{PNV}} \cap B_{\widetilde{PNV}}) = \widetilde{PNV}\ int\ (A_{\widetilde{PNV}}) \cap \widetilde{PNV}\ int\ (B_{\widetilde{PNV}})$

$d_1)\ \overline{PNV}\ cl(A_{\overline{PNV}} \cup B_{\overline{PNV}}) = \overline{PNV}\ cl(A_{\overline{PNV}}) \cup \overline{PNV}\ cl(B_{\overline{PNV}})$

e) $\overline{PNV}\ int(1_{\overline{PNV}}) = 1_{\overline{PNV}}$

$e_1)\ \overline{PNV}\ cl(0_{\overline{PNV}}) = 0_{\overline{PNV}}$

Proof: $(a), (b)\ and\ (c)\ are\ obvious\ and\ (c)\ follows\ from\ (a).$

(d) from $\overline{PNV}\ int(A_{\overline{PNV}} \cap B_{\overline{PNV}}) \subseteq \overline{PNV}\ int(A_{\overline{PNV}})$ and $\overline{PNV}\ int(A_{\overline{PNV}} \cap B_{\overline{PNV}}) \subseteq \overline{PNV}\ int(B_{\overline{PNV}})$

We obtain $\overline{PNV}\ int(A_{\overline{PNV}} \cap B_{\overline{PNV}}) = \overline{PNV}\ int(A_{\overline{PNV}}) \cap \overline{PNV}\ int(B_{\overline{PNV}}) \rightarrow (1)$

$On\ the\ other\ hand, from\ the\ facts\ \overline{PNV}\ int(A_{\overline{PNV}}) \subseteq A_{\overline{PNV}}$ and

$\overline{PNV}\ int(B_{\overline{PNV}}) \subseteq B_{\overline{PNV}}$,

we have $\overline{PNV}\ int(A_{\overline{PNV}}) \cap \overline{PNV}\ int(B_{\overline{PNV}}) \subseteq A_{\overline{PNV}} \cap B_{\overline{PNV}}$ and

$\overline{PNV}\ int(A_{\overline{PNV}}) \cap \overline{PNV}\ int(B_{\overline{PNV}}) \in \tau_{\overline{PNV}}$

We see that $\overline{PNV}\ int(A_{\overline{PNV}}) \cap \overline{PNV}\ int(B_{\overline{PNV}}) \subseteq \overline{PNV}\ int(A_{\overline{PNV}} \cap B_{\overline{PNV}}) \rightarrow (2).$

From (1) and (2) $\overline{PNV}\ int(A_{\overline{PNV}} \cap B_{\overline{PNV}}) = \overline{PNV}\ int(A_{\overline{PNV}}) \cap \overline{PNV}\ int(B_{\overline{PNV}})$

(a_1) - $(e_1)\ can\ be\ easily\ deduced\ from$ (a) – (e).

Definition 4.7:

The normalized Euclidean distance for pentapartitioned neutrosophic $sets\ A\ and\ B\ in$ $X = \{x_1, x_2, \ldots\ldots\ldots x_n\}$ is

$$\left(\frac{1}{n}\sum_{i=1}^{n}\left(T^{+}_{A_{PNV}}(x_i) - T^{+}_{B_{PNV}}(x_i)\right|\right)^2 + \left(T^{-}_{A_{PNV}}(x_i) - T^{-}_{B_{PNV}}(x_i)\right|\right)^2 + \left(C^{+}_{A_{PNV}}(x_i) - C^{+}_{B_{PNV}}(x_i)\right|\right)^2 + \left(C^{-}_{A_{PNV}}(x_i) - C^{-}_{B_{PNV}}(x_i)\right|\right)^2 + \left(G^{+}_{A_{PNV}}(x_i) - G^{+}_{B_{PNV}}(x_i)\right|\right)^2 + \left(G^{-}_{A_{PNV}}(x_i) - G^{-}_{B_{PNV}}(x_i)\right|\right)^2 + \left(U^{+}_{A_{PNV}}(x_i) - U^{+}_{B_{PNV}}(x_i)\right|\right)^2 + \left(U^{-}_{A_{PNV}}(x_i) - U^{-}_{B_{PNV}}(x_i)\right|\right)^2 + \left(F^{+}_{A_{PNV}}(x_i) - F^{+}_{B_{PNV}}(x_i)\right|\right)^2 + \left(F^{-}_{A_{PNV}}(x_i) - F^{-}_{B_{PNV}}(x_i)\right|\right)^2\Bigg)^{\frac{1}{2}}$$

5. Application of Pentapartitioned Neutrosophic Vague Sets in Health Care

We conducted a survey among the employers in a company. We have compiled a list of the most frequently encountered issues. Among the employers we have considered only four employers, let the employers be E1, E2, E3, E4 are denoted by the set E = {E1, E2, E3, E4} and the set of symptoms $S = \{Head\ ache, acidity, burning\ eyes$ and $back\ pain\}$. Let the set of health problems be $H = \{Stress, ulcer, vision\ problem\ and\ spinal\ problem\}$.

Table 1: Represents the employers and their symptoms

	Head ache	*Acidity*	*Burning eyes*	*Back pain*
E1	([0.4,0.9],[0.2,0.5], [0.6,0.8],[0.5,0.8], [0.1,0.6])	([0.2,0.7],[0.2,0.6], [0.5,0.8],[0.4,0.8], [0.3,0.8])	([0.1,0.9],[0.3,0.6], [0.7,0.9],[0.4,0.7], [0.1,0.9])	([0.1,0.7],[0.2,0.4], [0.3,0.8],[0.6,0.8], [0.3,0.9])
E2	([0,0.7],[0.3,0.5], [0.2,1],[0.5,0.7], [0.3,1])	([0.4,0.5],[0.1,0.5], [0.3,0.7],[0.5,0.9], [0.5,0.6])	([0.2,0.3],[0.4,0.6], [0.2,0.4],[0.4,0.6], [0.7,0.8])	([0.3,0.6],[0.5,0.6], [0.7,0.8],[0.4,0.7], [0.4,0.5])
E3	([0.1,0.7],[0.2,0.9], [0.3,0.5],[0.1,0.8], [0.3,0.9])	([0.3,0.9],[0.1,0.6], [0.2,0.6],[0.1,0.7], [0.4,0.9])	([0.2,0.5],[0.4,0.5], [0.3,0.7],[0.5,0.6], [0.5,0.8])	([0.3,0.9],[0,0.3], [0.4,0.7],[0.7,1], [0.1,0.7])
E4	([0.1,0.5],[0.3,0.9], [0.5,0.6],[0.1,0.7], [0.5,0.9])	([0.3,0.4],[0.2,0.7], [0.4,0.5],[0.3,0.8], [0.6,0.7])	([0.4,0.5],[0.5,0.7], [0.8,0.8],[0.3,0.5], [0.5,0.6])	([0.2,0.6],[0.4,0.8], [0.3,0.7],[0.2,0.6], [0.4,0.8])

Table 2: Represents related health problems

Using Definition 4.7 above to calculate the distance between each employer and each health problem with reference to the symptoms, we get the table below.

	Stress	**Ulcer**	*Vision problem*	*Spinal Problem*
Head ache	([0.3,0.5],[0,0.3] [0.4,0.7],[0.7,1], [0.5,0.7])	([0,0.6],[0.4,0.6], [0.3,1],[0.4,0.6], [0.4,1])	([0.2,0.2],[0.4,0.8], [0.4,0.5],[0.2,0.6], [0.8,0.8])	([0.2,0.8],[0.3 ,0.6], [0.2,0.6],[0.4, 0.7], [0.2,0.8])
Acidity	([0.3,0.5],[0.4,0.5],[0.7,1],[0.5 ,0.6], [0.5,0.7])	([0.2,0.6],[0.3,0. 4], [0.5,0.8],[0.6,0.7], [0.4,0.8])	([0.1,0.5],[0.2,0.4], [0.8,0.9],[0.6,0.8], [0.5,0.9])	([0.1,0.5],[0.4 ,0.5], [0.3,0.9],[0.5, 0.6], [0.5,0.9])
Burning eyes	([0.2,0.8],[0.4,0.7],[0.6,0.7],[0 .3,0.6], [0.2,0.8])	([0,0.8],[0.1,0.5], [0.2,0.5],[0.5,0.9], [0.2,1])	([0.1,0.7],[0.4,0.6], [0.3,0.9],[0.4,0.6], [0.3,0.9])	([0.5,0.7],[0,0 .3], [0.5,0.9],[0.7, 1], [0.3,0.5])
Back pain	([0.1,0.7],[0.3,0.4],[0,6,0.7],[0 .6,0.7], [0.3,0.9])	([0.5,0.5],[0,0.2], [0.4,0.7],[0.8,1], [0.5,0.5])	([0.2,0.6],[0.3,0.4], [0.5,0.8],[0.6,0.7], [0.4,0.8])	([0.1,0.7],[0.5 ,0.6], [0.3,0.9],[0.4, 0.5], [0.3,0.9])

Table 3: Employers vs Health Problems

	Stress	***Ulcer***	***Vision problem***	***Spinal Problem***
E1	0.3317	0.2280	0.5196	**0.1949**
E2	0.3256	**0.2236**	0.4171	0.2932
E3	0.2569	0.2720	**0.1789**	0.4494
E4	0.3464	0.3795	0.3	**0.1897**

Table 3 shows that the Normalized Euclidean distance method yields the final results for four employers' health problems.

(i) The shortest value of E1 $is\ 0.1949$ and therefore E1 faces spinal problem.
(ii) The shortest value of E2 $is\ \ 0.2236,$ indicating faces ulcer.
(iii) The shortest value of E3 $is\ 0.1789$ therefore E3 suffers from vision problem.
(iv) The shortest value of E4 $is\ 0.1897$ and therefore E4 has a spinal problem.

Conclusion

In this article, we have defined the term "pentapartitioned neutrosophic vague set," examined its features and operations, and provided some instances. Pentapartitioned Neutrosophic vague sets and topological spaces were employed, and a novel notion known as "Pentapartitioned Neutrosophic vague topological spaces" was developed. Its characteristics and potential uses in medical diagnosis were also examined.

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