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Applications of Graph Theory to Urban Drainage Networks Layouts

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Abstract
The Recent studies have shown the practicality and usefulness of decentralized paradigms in the context of designing Urban Drainage Networks for meeting present and future objectives for sustainability and resilience. However, there isn't a single, comprehensive framework that can be used right away. In order to list all conceivable combination of outlet candidates and the related layouts in the Urban Drainage Networks, this is achieved by combining a unique deterministic graph-theory-based layout generator with a deterministic multi-objective combinatorial optimization. By dividing a big, completely centralised network into many smaller branches, the strain on the centralized pipes is redistributed, leading to more economical alternatives. Urban Drainage Network layout characteristics including the length area index (LAI), average reliability index (ARI), and negative slope index (NSI) are optimized using a novel hybrid optimisation model called the Archimedes Search and Rescue Optimisation Algorithm (ASROA).The ASROA model is a conceptual combination of the search and rescue optimization method (SAR) and the basic Archimedes optimisation algorithm (AOA). The fascinating physics concept known as Archimedes' Principle provided the impetus for developing this AOA model. It simulates the notion that an object's buoyant force, whether fully or partly submerged, is proportional to the weight of the displaced fluid. The Search and Rescue (SAR) optimization technique is presented in this study as a solution to restricted engineering optimization problems. This metaheuristic algorithm simulates human exploration and search and rescue activities. This layout parameter optimization is significantly reducing the computational effort.

Keywords: Urban Drainage Networks (UDNs),ASROA,Length Area Index (LAI), Average Reliability Index (ARI), Negative Slope Index (NSI),Graph Theory

Introduction

One of the most important infrastructures in each community, urban drainage networks (UDNs), is to blame for both structural (like blockage-induced failure) and functional (like blockage-induced failure) problems [1]. However, because to the numerous links between UDNs, assessing construction, maintenance, administration, and operational management in isolation is not feasible. In addition, the aging of natural resources, urbanization, population increase, and climate change have the potential to seriously impair urban water systems, particularly UDNs, and to present serious health risks. Resilience enters the scene at this point [2-4]. Resilience is the system's capacity to minimize the consequences of a failure and quickly and safely recover from any unusual loading demands. UDNs must thus be prepared to manage both typical and unforeseenly high loading conditions.While most study has concentrated on the resilience of sewage network rehabilitation, the effect of the system's topological layout on resilience at the design stage has been disregarded [4]. Typically, sewage systems are built with a centrally situated network of pipes that are linked to a single treatment facility or outlet. However, centralized UDNs have come under heavy fire because they impose large financial burdens on stakeholders and governments, particularly in poor nations [5–6]. A few studies have highlighted the issues with centralised infrastructure that go beyond economic ones, including ecological and environmental issues, climate change vulnerability, and developing countries' limited ability to quickly adjust.

Sewage overflow prevention is essential to reducing public health concerns and safeguarding the environment from contaminated water. Surface waterways may be exposed to partly or fully untreated sewage as a result of sanitary sewer overflows (SSOs) or combined sewer overflows (CSOs) [7]. Because untreated sewage

overflows contain suspended particles, toxics, nutrients, debris, and other contaminants, they pose a serious threat to the quality of the water. Sewer pipe capacity may be exceeded by rainfall-derived input and infiltration (RDII) from rainy weather flows, leading to an SSO. Equipment malfunctions, undersized sewer pipes, power outages, silt buildup, pipe breaks, leaking manholes, offset joints, debris and grease blockages in the sewer conduits, and other issues can also result in SSOs.When an SSO occurs, sewage overflows onto parks, streets, and waterways [8–9]. Furthermore, there is a chance that this sewage will back up into basements, posing a health risk to anyone who comes into touch with it and causing property damage [10–11]. Under the jurisdiction of the Clean Water Act, which Congress authorized, the Environmental Protection Agency (EPA) has established wastewater regulations for the industry and started pollution management programs in response to growing public expectations for high-quality services [12]. To ensure that wastewater flows are reliably conveyed without producing overflows or backups, comprehensive modeling and analysis of these sewage systems is required in order to propose excellent, inexpensive solutions for increasing system integrity and performance.

Planning and building more effective sewage collection systems often make use of drainage network simulation models [13–14]. Adding more storage volume, increasing pumping capacity, expanding conduit capacity (larger interceptors), adding new sewer pipes or treatment capacity, increasing pumping capacity, and maximizing the use of the system's current storage by utilizing real-time operational controls are among the options for system improvement that are routinely assessed [15–16]. The difficult aspect is figuring out which update, or which improvements together, will address the flooding problem most efficiently for the least amount of money. The same update strategy is not beneficial for all collecting systems [17–18]. Different systems respond to remedial actions differently in terms of cost and efficacy. The present evaluation procedure that is used to choose an upgrade choice is time-consuming and often results in neither the optimal not the most cost-effective update for collecting systems [19]. This method assesses the hydraulic performance of the present system for several design options (modifications) under various loads and operating situations using a drainage network simulation model [20].

The primary contributions of this study are as follows: This study presents a methodology for creating and optimising decentralisation scenarios concerning the layout of sewage configurations.

- It builds a graph from the primary input data enabling graph analysis for (de)centralized layout generation
- The produced Pareto-front solutions are then converted to the SWMM input files.
- The resulting layouts on Pareto-front are hydraulically designed, and are then evaluated for resilience and construction costs/life cycle costs.

The following pattern guided the arrangement of the chapters: Chapter 1 provides a basic introduction; Chapter 2 provides the theoretical background of the literature review conducted for this research work; Chapter 3 provides an overview of the proposed methodology; Chapter 4 provides the use of the proposed algorithm; Chapter 5 summarises the experimental results; and Chapter 6 presents the research work conclusions.

Recent research work: A brief review

Numerous research studies on writing that used multi-objective algorithms based on graph theory and urban drainage network layouts have already been carried out from a range of perspectives and methodologies. The works that have been reviewed are listed below.

A unique dynamic, multi-objective optimisation method has been presented by Giudici et al. [21] to improve the sustainability of tiny islands by utilising renewable energy sources. The three main contributions of our technique are (i) collaborative optimisation of system design and operations, (ii) multi-objective optimisation to investigate trade-offs between potentially incompatible aims, and (iv) dynamic modelling of desalination plant operations. We evaluate our method on the real case study of the Italian island of Ustica, using a comparative analysis with a traditional non-dynamic, least equal optimisation methodology.

An integrated Graph Theory-Based Bi-Level Water Supply Planning Model, known as GraBiL, was developed in this study as a methodological response to the problem of statistical interrelationships of WECN and hierarchical decision-making dilemma in regional-scale water network planning, as presented by Chenet al. [22]. Compared to the current bi-level coding and graph theory-based techniques, the GraBiL model has proven more effective in solving the spatial layout optimisation challenge in a two-level decision-making dilemma. It was considered that there are two tiers of competing objectives, for example, maximising energy savings and minimising overall system costs. Fuzzy uncertainty associated with water load are quantified. In an experiment, Najafiet al. [23] improved resilience by increasing consumers' access to water and power in the wake of natural disasters. Microgrids can provide the electricity needed to re-connect disconnected loads in distribution networks when they are run correctly. Within the proposed interactive framework, a stochastic power management programme for microgrids was created that accounts for the reliability of local loads during

emergencies in addition to predicting the amount of energy that may be moved to distribution networks. “During an emergency, each microgrid provides a list of bid-quantity energy blocks to the distribution system operator (DSO).

In order to reduce load variation and hydraulic imbalance, Ding, et al. [24] proposed a dynamic programming approach based on graph theory for the deployment of pipeline network architectures and the selection of energy station sites. The regional building load was dispersed by using the kernel density approach while picking the locations of energy stations. To increase the energy supply range of the energy station with the lowest load variation rate as the aim, a 0-1 dynamic programming approach was proposed. A graph theory-based improved Prim approach was developed to forecast the pipeline network architecture.

After experimenting to optimise the UDS layout, Bakhshipouret al. [25] propose simplified cost and structural resilience indices in this work, which can be used as heuristic parameters. Graph connection data is all that these indexes employ, and it uses a lot less computing power than hydraulic simulation. By using simpler objective functions for optimisation, the viable search space's complexity and the number of blind searches were significantly decreased. To demonstrate the application and advantages of the recommended strategy, a real case study in the southwest Iranian city of Ahvaz was looked into. In terms of generating robust and realistic UDNs and requiring less processing power, the suggested design seems promising.

A unique approach that combines a combinatorial optimisation process and an improved hybrid community detection algorithm has been tested by Shao, Y. et al. [26]. During the node clustering phase, the hybrid approach, which was based on the improved modularity index, allows for the quick development of partition solutions with a more uniform distribution of water demand and reduced boundary pipe widths. Then, utilising a three-step optimisation process that comprises preliminary hydraulic analysis, searching for a suboptimal solution, and multi-objective optimisation in the partition dividing phase, the location of flow metres and isolation valves in WDNs was swiftly and successfully resolved.

A novel complex networks analysis-based method for assessing computational complexity and efficiency of water quality in a WDS was developed and extensively tested by Sitzenfrei[27] (R2 values in comparison with state-of-the-art nodal water qualities are reached in the range of 0.95 to 0.95). The recommended model was successfully used in a design study to identify the design solutions that exceeded water quality norms, with an accurate identification rate ranging from 96% to 100%. The computational efficiency was discovered to be 4.2e-06 lower than that of contemporary” models.

Background of recent research work

Most authors of research on optimum UDNs have focused on different approaches to optimally size UDN pipes, with little emphasis on designing and refining UDN topological layouts. Furthermore, a great deal of these research solely considered centralised design. These acts, as previously stated, do not promote resilience and sustainability methods or long-term goals. In steep areas, designers often produce and optimise layout configurations based on the street's terrain and the related steepness. They do this by tracking the natural ground slope gravitationally to a specific discharge point.Throughout the process, engineering decisions have a critical role in determining the degree to which a near-optimal solution is produced. This difficulty is further compounded when large-scale catchments and decentralisation considerations are included. There isn't a systematic or all-encompassing framework that can generate the optimal decentralised arrangements for these terrains, notwithstanding the possibility of success and maybe even subjective success. “Several strategies have been proposed for decentralized/hybrid urban water management systems, including reducing the degree of centralization (DC) or splitting off a portion of a centralised tree-like network into a cluster of trees. On-site runoff controls made available by low-impact building, best management practices, green infrastructure, and water-sensitive urban design are examples of alternative hybridization strategies.

Proposed methodology

By creating a graph from the major input data (i.e., street maps and the impervious layer), graph analysis may be used to produce (de)centralized layouts in combination with multi-objective combinatorial optimisation. Next, the generated Pareto-front solutions are transformed into the input files for the storm water management model. Following a thorough pre-screening process using design rainfall events, the resultant Pareto-front layouts are hydraulically developed and then assessed for resilience and life cycle/construction costs.Figure 1 illustrates how the suggested technique is configured.

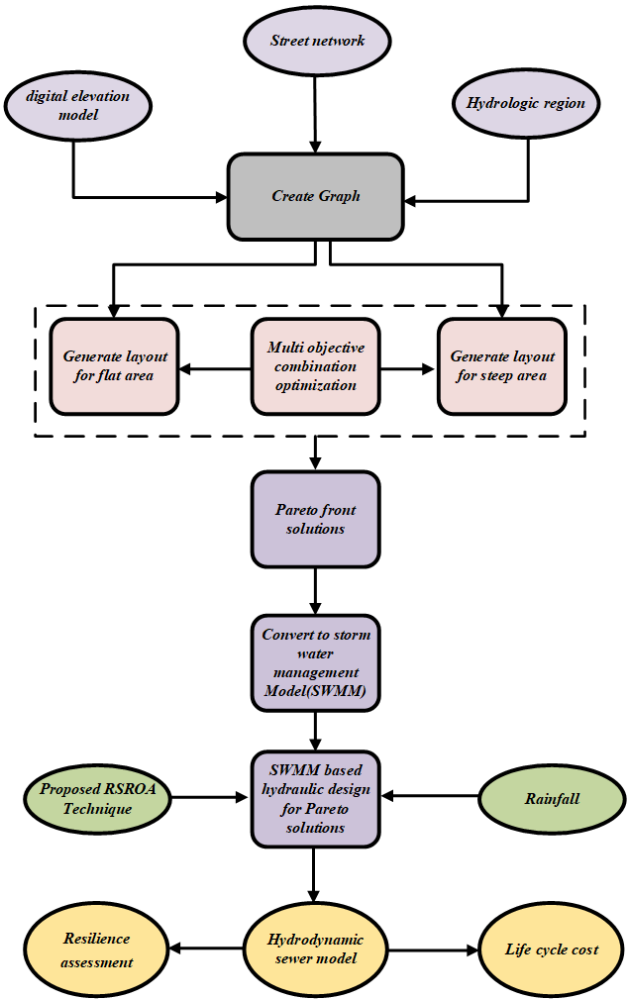


Fig 1: Configuration of proposed methodology

Graph representation of urban drainage network layouts

To generate the initial layouts and base graphs, the first open street maps of the locations under examination are acquired. The vertices of these maps are intersections or road junctions, while the edges are streets or segments. This spatial network is then used to collect data from the Thiessen polygon-formed sub-catchment (such as impervious layers) at each road crossing. After the street network is mapped to the region's digital elevation model (DEM), which provides the ground elevation data for each node/junction, the base graph (original layout) of the area is eventually created. In this way, the architecture of urban drainage networks (UDNs) may be explained using the mathematical concept of graph theory.Imagine a graph G with many vertices (such outlets, storage areas, and manholes) and many links (like conduits, weirs, and pumps) or edges. Furthermore, the graph edges may use various weighting factors, including pipe sizes, lengths, and hydraulic characteristics. The first metric in this research is the shortest route between nodes i and j, where the cost, expressed as the sum of (positive) edge weights, is lowest. The edge lengths are utilised as weighting factors in the shortest route method, which is used during the layout-generating step.

Another notion employed in this research is the degree of a node, which indicates the amount of edges connecting to a node. In a directed graph, where each edge points in a certain direction, each node is represented by two distinct degrees: the outdegree, which shows the number of edges entering the node, and the indegree, which shows the number of edges leaving the node.

The next topological network parameter used is edge betweenness centrality (EBC), which counts the number of shortest paths connecting each pair of nodes in a graph. Since every node in an urban drainage network is linked to at least one output node, the shortest path between any two nodes may be found. We next count the number of shortest routes that connect each input node (i) to the output node (j). Moreover, in contrast to the traditional method of calculating EBC, which increases the counts by 1 if an edge e is a segment of the shortest route, we include the contribution runoff area Ri from each (inlet) node (i) towards the EBC values. The total impervious area at each edge may be calculated more easily as a result.

The expression for this measurement, known as "runoff edge betweenness centrality" EBC_e^R , is as follows:

$$EBC_e^R = \sum_{j=1}^{ON}\sum_{i=1}^{IN}\sigma i, j(e) \times R_i \quad (1)$$

Layout generator for centralized and decentralized systems

Establishing an optimal and cost-effective UDNs design requires careful consideration of the layout of an Urban Drainage Networks, which shows the location and arrangement of pipes, junctions, manholes, and the direction of sewage flow.Our approach is not appropriate for the perfectly flat terrain, where we assume that there is no effective ground slope to determine the directions.

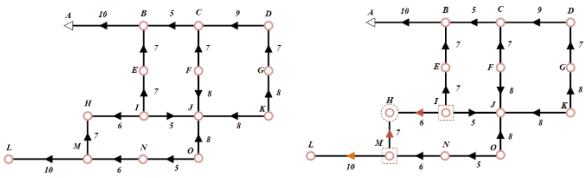
Direction modifier module (for steep terrains)

The primary goal of this module is to reverse the flow of edges, or pipes, on steep terrain. The primary driving force is to maintain the existing orientations while changing certain problematic ones and enforcing negative slopes, which need larger excavation volumes. This tactic may be useful, particularly over steep, uneven terrain. As a matter of fact, steep terrain in certain areas indicates patterns for tracking, following, and constructing gravity pipelines; yet, pipes still need to be built against the natural slope of the land. These directions must thus be adjusted to point in the direction of the potential outlet.

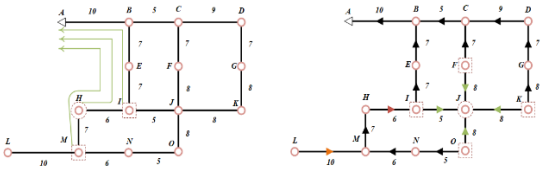
A directed looping graph is produced after this process is finished (i.e., there are no more converged nodes), and it is then simple to distinguish between the slopes of the edges, or directions, that are positive or favourable. This makes it possible for us to subsequently identify the layout that, while still permitting decentralisation with various outlet configurations, minimises the total negative slopes. This approach (but all possible sewer manholes) is much more beneficial when not all possible sewer linkages are included in the design creation. Pipes with negative slopes may be readily identified and avoided, requiring bigger construction volumes in order to produce an inexpensive loop-free drainage network.

Layout generator module for steep and flat areas

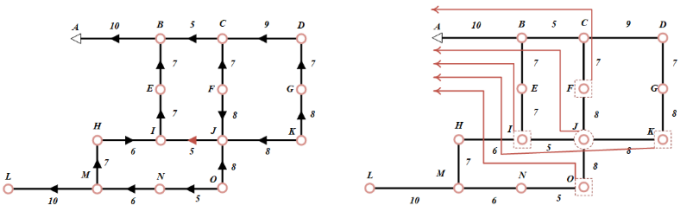
For the creation of sewer systems in steep terrain, the flow directions on the base graph must be adjusted when there is only one exit. Thus, if just the centralised dendric arrangement (one outlet) is needed, the edges in the directed looping graph that generate the loops may be readily severed from their upstream ends. However, the transformation of such a directed base network into a decentralised one may now begin if the number and location of outlet options are given. The decentralisation approach for flat terrain is comparable to that for steep terrain in several aspects. Apart from that, we are left with no edge flow directions since there is no effective ground slope.



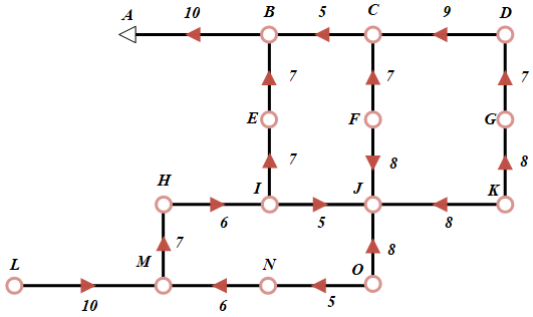
1. beginning the procedure with a directed base graph, where the ground's natural heights dictate the flow directions Finding the converged node (node H) and any predecessors (nodes I and M) after flipping or reversing the direction of the edge that connects to the dead-end node (edge ML).



4. Using the shortest route (edge IH to HI) to reverse the red-colored arrows (edges) that emerged in separate directions in step (3), and then identifying the next convergent node (node J) and its predecessors. 3. Attempting to apply the lengths as weight shortest route method across the unconstrained base network from the converged node and its predecessors to the output (node A) (nodes F, I, K and O)

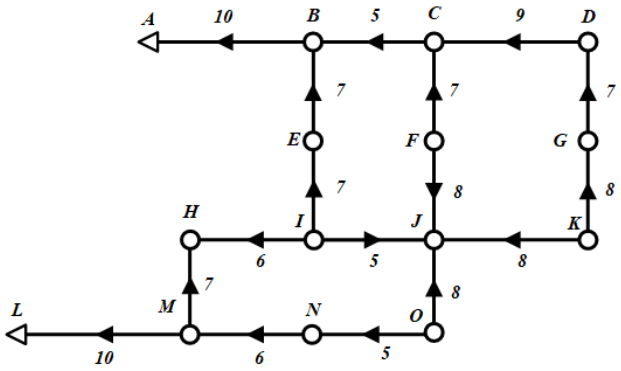


5. Repeating step (3), 6. choosing the quickest route among all the red arrows in step (4) to reverse those that appeared to be going in the opposite directions in step (5) (edge IJ to JI).

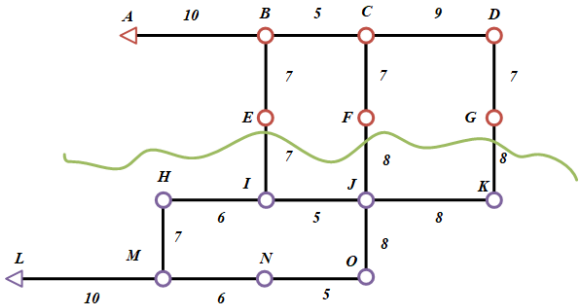


7.Stopping the process when there are no more converged nodes.

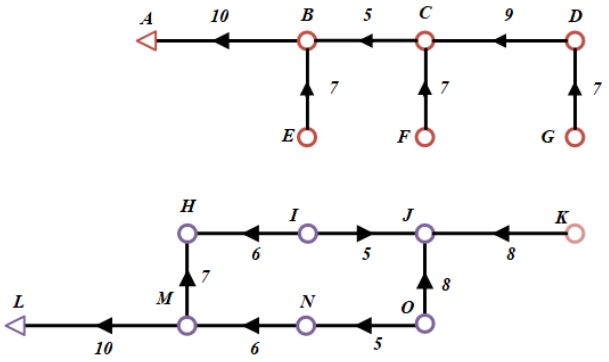
Steep Terrain



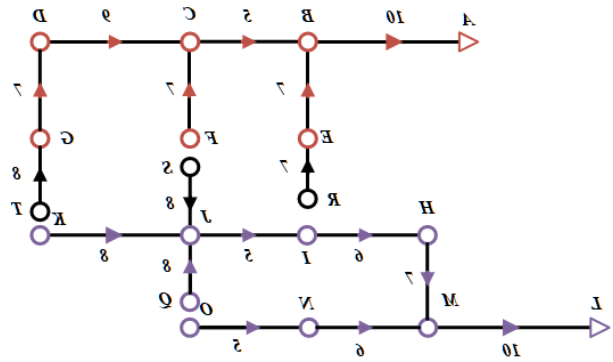
(1) starting the process with a directed base graph with two outlet options and flow directions based on ground elevations (outlets A and L)



(2) depending on their shortest/closest distance to each outlet, assigning the nodes to the outlets across the undirected base network (lengths as weight)

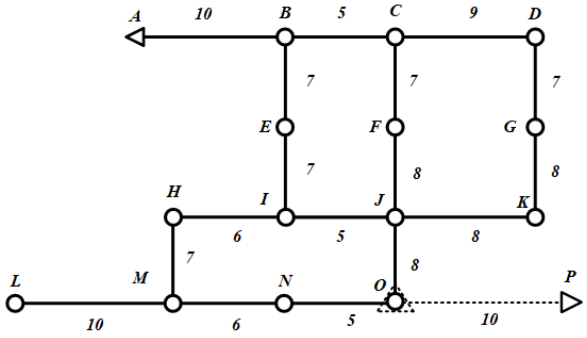


(3) Deleting the joint edges (edges GK, FJ, and EI) that are a component of both partitions and overlaying the left directed base graph to take use of the topography

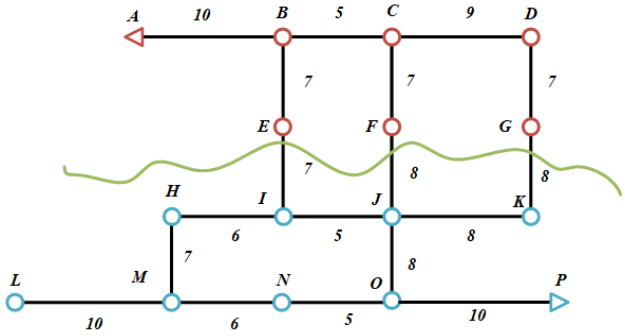


(4) Applying a direction modifier to each base graph that has been divided, adding and removing the joint edges (edges RE, SJ, TG, and the edge QJ), and simultaneously adding additional nodes (nodes Q, R, S and T)

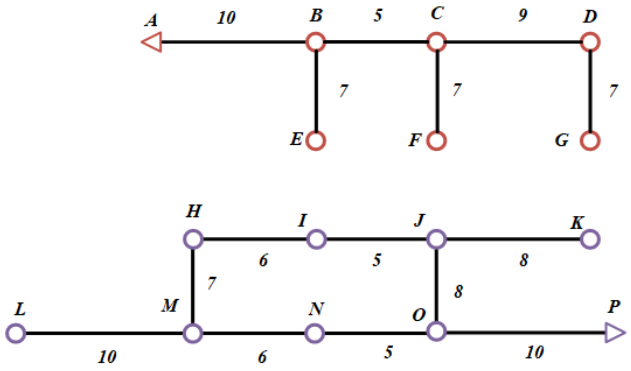
Flat terrain



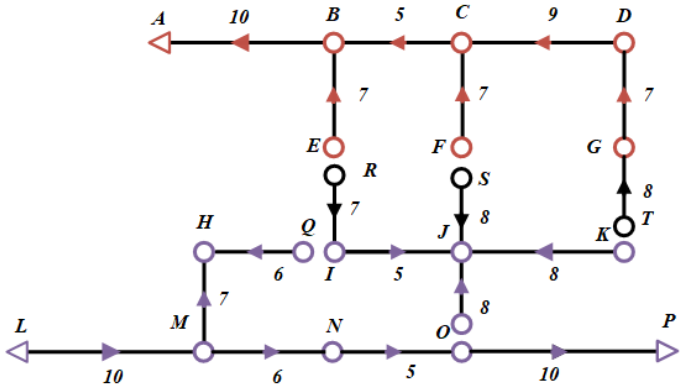
(1) starting the process with a base graph that is undirected and has two outlet possibilities (outlets A and O); outlet O is transported to P and connected to the method by a single edge (a pipe) (edge OP)



(2) depending on their smallest route to each outlet, allocating the nodes to the outputs across the undirected base network (lengths as weight)



(3) Eliminating the junction edges on both partitions (edges GK, FJ and El)



(4) Running the shortest path method with the lengths as weights, adding and removing the joint edges (edges RI, SJ, TG, and edge QH), and simultaneously adding additional nodes (nodes Q, R, S and T)

Layout objective functions

It is feasible to investigate the design costs (and, therefore, the optimal layout) of all generated layout combinations once certain features, such as pipe diameters, lengths, and excavation depths, are known ahead of time. Except for pipe lengths, none of the entrance modes are identified during the planning stage. Furthermore, if every possible arrangement were hydraulically built in order to calculate costs, there would be an enormous computational demand [28]. Consequently, computational efficiency may be significantly increased by eliminating a set of sewer layouts that perform badly in comparison to the other solutions before submitting it to the hydraulic design and optimisation. Consequently, three simple and generalised layout cost functions are presented that function with respect to the readily attainable surface slope, cumulative runoff area, and edge (pipe) lengths.These functions are the length area index (LAI), average reliability index (ARI), and negative slope index (NSI). The following provides an explanation of each:

Length area index (LAI)

The expense of a layout in proportion to the length and slope flow rate function of each pipe is taken into consideration by Walters and Smith (1995) in their widely used layout goal functions in the area. At this stage of the planning process, however, none of the hydraulic properties, such as flow rates, are known. In this instance, volumetric flow discharges may be estimated using proxies, the total contributing impervious areas [29]. This measurement, LAI, provided in Eq. (2), illustrates a correlation between the accumulated runoff contributing area of each edge $R_{cum,e}$ and the edge lengths L_e.

$$LAI = \sum_{e=1}^{e=E} L_e \sqrt{R_{cum,e}} \tag{2}$$

Where, LAI is denoted as the length area index [m2], L_e is denoted as the edge length [m], and $R_{cum,e}$ is denoted as the cumulative runoff contribution area of edge e.

Furthermore, the graph connect matrix makes it simple to determine the length of every edge. In this paper, we present a unique graph-theory based method for calculating the total runoff contribution with each edge/pipe or $R_{cum,e}$, for which Eq. (1) is suitable to sewage tree-shaped networks. The obtained values for EBC_e^R in Equation (1) correlate to $R_{cum,e}$ in Equation (2), which represents the total contributing impervious areas. This straightforward graph theory relationship leads to a quick approach to ascertain the total runoff area for every pipe.

Average reliability index (ARI)

The creators of this index, Haghighi and Bakhshipour (2016), said that the average dependability of sewage collecting systems could be estimated by looking at the population impacted by a blockage and the impact it had on the region upstream. Put another way, when system reliability increases, the effect of restrictions on the individuals at upstream levels decreases. Nonetheless, this metric is aligned here since it was intended to function in connection with the network of stormwater conveyance systems rather than public sewage systems based on the cumulative sewage output [30].In light of this, Eq. (3) may be used to determine the reliability index of each individual edge (pipe) RI_e as a function of both the total runoff area R_{total} and the cumulative contributing runoff of each edge (pipe), denoted as $R_{cum,e}$. Higher pipe dependability is associated with lower levels of $R_{cum,e}$.

$$RI_e = 100 \times \left(1 - \frac{R_{cum,e}}{R_{total}}\right)(\%) \tag{3}$$

Where, RI_e indicates the reliability index of every single edge (pipe), $R_{cum,e}$, indicates the cumulative contributing runoff of every edge, and R_{total} indicates the total runoff area.

The average of all the edges is used to extend the RI_e of each single edge to the rest of the network (E). This measurement (Eq. (4)) is also known as the average reliability index (ARI) [31]. Different runoff distribution and progression toward the outlet are triggered in this context by generating varied (de)centralized contribute to ensuring (topological layouts). In order to focus on the optimal runoff area distribution for every created structure/layout, ARI is used as another objective function.

$$ARI = \frac{\sum_{e=1}^{e=E} RI_e}{E} \tag{4}$$

where ARI is the average reliability index for the entire network, RI_e is the reliability index (ha/ha) for each pipe or edge, and E is denoted as the total number of edges (pipes).

Negative slope index (NSI)

We were able to effectively change the directed base graph's flow directions for steep terrains. As a result, we are able to calculate the slope of each edge and identify those with favorable/positive or unfavorable/negative slope for the flow direction [32]. The slope of a pipe or edge e is calculated as the length L_e divided by the sum of its upstream and downstream nodes' ground elevations (UE_e and DE_e, respectively). In Eq. (5) the slope index (SI) is described.

$$SI_e = (UE_e + DE_e) / L_e \tag{5}$$

Where, SI_e is the slope index of edge e [m/m], UE_e is the upstream elevation of edge e [m], DE_e is the downstream elevation of edge e [m], and L_e is the length of edge e [m].

Because greater excavation volumes are produced by the hydraulic pipe sizing, the associated edge's slope is negative (unfavourable slope) if the surface level of the upstream node/manhole is less compared to the downstream node/manhole. Therefore, through layout improvement in steep terrain, maximising the net negative slope index of all edges, or NSI (experiencing negative values), is a very important objective function. In Eq. (6), the NSI computation is displayed.

$$NSI = \sum_{e=1}^{e=E} SI_e \left(SI_e < 0 \right) \tag{6}$$

Where, NSI is the (total) negative slope index [m/m], and SI_e is the slope index of edge e [m/m]

Optimization of layout solutions

The degree of centralisation (DC) for the given layout alternatives is first determined using a modified and generic metric based on the total number of intake nodes and the number of chosen exits, as described in this Section. After that, a brief description of multi-objective optimisation is given, which is the process of selecting the best possible arrangement scenarios.

We must first determine how disconnected a system is, or how much water is being discharged to different outlets, as the purpose of this research is to design both centralised and decentralised topological layouts [33]. It is challenging to evaluate and compare the degree of centralisation with several other networking (based on the amount of potential and selected outlets) since the literature-based index clearly illustrates the notion of decentralisation for each network. In light of this, this research proposes a new metric that may be used as a generic tool to expand and generalise the DC index (degree of centralisation) across all networks, including stormwater and sewage systems.The number of (selected) output nodes (ONs, up to IN-1) and the total number of intake nodes (IN) have a linear relationship, according to Eq. (7):

$$DC = 100 \times \left(1 - \frac{\log_{10}^{ON_s}}{\log_{10}^{IN}} \right) (\%) \tag{7}$$

This formula implies that DC equals 0% (complete decentralisation) if every intake node serves as an output node; however, in real life, this is not possible. Furthermore, only a completely centralised single output node can achieve 100% DC.

Because of the deterministic behaviour of the suggested layout generating method in this research, there is only one unique sewer layout choice for every scenario of potential outlet placement. It is necessary to look at all possible combinations of outlet positions (DC) in order to determine the optimal layout with a certain outlet location or locations and the right number of outlets. In a (multi-objective) combinatorial optimisation, the only variable used to make decisions was the quantity of possible outlet candidates. After the layout target functions were established for each outlet arrangement scenario, the Pareto front was created by minimising LAI and maximising ARI for flat regions and minimising LAI and maximising ARI and NSI for steep areas. These Pareto-front solutions are then applied to the hydraulic design.

Hydraulic pipe sizing

The UDNs (urban drainage networks) must be designed with consideration for a number of constraints, including the telescopic pattern, minimum cover depth, maximum excavation depth, and minimum and maximum slope, in order to guarantee that the stormwater collection system satisfies all essential hydraulic and technical requirements. Three primary components make up the optimisation framework used in this hydraulic design method.
An adaptive algorithm that can adapt to the optimization's limitations is used in the urban drainage networks

optimisation process in stormwater collecting systems. Furthermore, a module that makes use of the UDNs graph's connection matrix—which can be derived from the planned layout—is used to narrow the scope of the optimisation search and save computing power. An optimisation engine that uses metaheuristics, such simulated annealing, is then used to solve the optimisation issue. The efficient and effective optimisation of the UDNs to satisfy the essential hydraulic and technical criteria is made possible by this combination of components.

Resilience assessment

When deciding on the final network, resilience of the UDNsis just as important as design costs (CC for steep terrain and (LCC for flat area)), particularly in light of continuing climate change effects. To thoroughly capture the respect to sustainable development the functional resilience response of the UDNs, the performance of the constructed (de)centralized structures is assessed during low-, medium-, and high-intensity design storms. For the steep case study and the flat case study, respectively, a cluster of storm occurrences with intervals of 5, 20, and 50 years is chosen. A hydraulic set of performance (HPI) is displayed in Eq. (8) to satisfy this demand

$$HPI = 100 \times \left(1 - \frac{V_{flooding}}{V_{runoff}} \right) (\%) \tag{8}$$

Where, HPI is denoted as the hydraulic performance indicator, $V_{flooding}$ is denoted as the total ponded flood volume [m3], and V_{runoff} is denoted as total runoff volume [m3].

Proposed ASROA Technique

The basic Archimedes optimisation algorithm (AOA) and the search and rescue optimisation algorithm (SAR) are conceptually combined in the ASROA model. The intriguing Archimedes' Principle is a rule of physics that served as motivation for the creation of this AOA model. It mimics the idea that the buoyant force applied to an object—whether submerged entirely or perhaps partially—is proportionate to the weight of the displaced fluid. This paper presents the Search and Rescue (SAR) optimisation approach for solving limited engineering optimisation issues. This metaheuristic algorithm mimics how people might explore and conduct rescue and search missions. This optimisation of the layout parameters helps to greatly lessen the computational load.

Archimedes optimization algorithm (AOA)

The AOA algorithm is a population-based algorithm. In the proposed method, the submerged objects are members of the population [34]. Like other population-based metaheuristic algorithms, AOA starts the search with an initial population of objects (candidate solutions) that have random volumes, densities, and accelerations. At this phase, the fluid's location is randomly assigned to each item. AOA evaluates the fitness of the original population and then iterates until the termination condition is satisfied. AOA updates each item's volume and density after each repetition. An object's acceleration varies according on whether it collides with any other adjacent objects. An object's updated density, volume, and acceleration define its new purpose. Below is a thorough mathematical description of the AOA phases.

The mathematical formulation of the AOA method is introduced in this section. AOA is theoretically regarded as a global optimisation technique as it encompasses both the exploration and exploitation phases.

Step 1: Initialization

Initialize the positions of all objects here the input is length area index (LAI), average reliability index (ARI), and negative slope index (NSI)

Step 2: Random generation

The initialization procedure generates input parameters randomly using the following Equation,

Step 3: Fitness Function

Evaluate the fitness values of initial solution candidates as in Equation

$$O_i = lb_i + rand \times \left(ub_i - lb_i \right); i = 1, 2, ..., N \tag{9}$$

Where, O_i is denoted as the i th object in a population of N objects. lb_i are denoted as the lower bounds ub_i are denoted as the upper bounds of the search-space, respectively

Initialize volume (vol) and density (den) for each i th object

$$den_i = rand$$

$$vol_i = rand \tag{10}$$

where $rand$ is indicates a D dimensional vector which randomly generates number between [0, 1]. And finally, initialize acceleration (acc) of i th object

$$acc_i = lb_i + rand \times \left(ub_i - lb_i\right) \tag{11}$$

Step 4: Update densities, volumes

The density and volume of object i for the iteration t + 1 is updated

$$\begin{aligned} den_i^{t+1} &= den_i^t + rand \times \left(den_{best} - den_i^t\right) \\ vol_i^{t+1} &= vol_i^t + rand \times \left(vol_{best} - vol_i^t\right) \end{aligned} \tag{12}$$

where vol_{best} and den_{best} are indicated as the volume and density associated with the best object found , and rand is indicated as the uniformly distributed random number

Step 5: Transfer operator and density factor

After some time has passed since the first collision, the objects attempt to attain an equilibrium condition. With the aid of the transfer operator T F, which changes search from exploration to exploitation, this is done in AOA.

$$T\,F = exp\left(\frac{t - t_{max}}{t_{max}}\right) \tag{13}$$

Where, transfer T F increases gradually with time until reaching 1. Here t and t_{max} are denoted as the iteration number and maximum iterations, respectively

$$d^{t+1} = exp\left(\frac{t_{max} - t}{t_{max}}\right) - \left(\frac{t}{t_{max}}\right) \tag{14}$$

where d^{t+1} diminishes with time, allowing convergence in a previously discovered suitable region

Step 6.1: Exploration phase

When an object collides with another, choose a random material (mr) and change the object's acceleration for iteration $t+1$.

$$acc_i^{t+1} = \frac{den_{mr} + vol_{mr} \times acc_{mr}}{den_i^{t+1} \times vol_i^{t+1}} \tag{15}$$

Where, den_i is denoted as the density, vol_i is denoted as the volume and acc_i is denoted as the acceleration of object i. Where acc_{mr} is denoted as the acceleration, den_{mr} is denoted as the density and vol_{mr} is denoted as the volume of random material.

Step 6.2: Exploitation phase

If TF > 0.5, there is no collision between objects.

update object's acceleration for iteration t + 1

$$acc_i^{t+1} = \frac{den_{best} + vol_{best} \times acc_{best}}{den_i^{t+1} \times vol_i^{t+1}} \tag{16}$$

Where, the acceleration of the best object is denoted as the acc_{best}

Step 4.3: Normalize acceleration

To calculate the change of percentage

$$acc_{i-norm}^{t+1} = u \times \frac{acc_i^{t+1} - min(acc)}{max(acc) - min(acc)} + l \tag{17}$$

where u and l, which are set to 0.9 and 0.1, respectively, are the normalisation ranges. The percentage of steps that each agent isalter is determined by the $acc_{i,norm}^{t+1}$. The acceleration value islarge if the item i is far from the global optimum, indicating that it is in the discovery phase; otherwise, it is in the exploitation phase. This exemplifies the transition from the exploration phase to the exploitation phase of the search. In a typical situation, the acceleration factor starts off with a high value and gets smaller over time. This enables search agents to move away from nearby solutions and toward the greatest global solution.

Step 7: Update position

If T F ≤ 0.5 (exploration phase)

The i th object's position for next iteration $t+1$

$$x_i^{t+1} = x_i^t + C_1 \times rand \times acc_{i-norm}^{t+1} \times d \times \left(x_{rand} - x_i^t\right) \tag{18}$$

where a constant called $C_1 = 2$

$$x_i^{t+1} = x_{best}^t + F \times C_2 \times rand \times acc_{i-norm}^{t+1} \times d \times \left(T \times x_{best} - x_i^t\right) \tag{19}$$

where a constant called $C_2 =$ 6. T is defined by $T = C_3 \times TF$ and rises over time while being directly proportional to the transfer operator. T initially subtracts a fixed percentage first from best possible position and grows with time in the range. Starting with a low percentage results in a significant gap between the optimal position and the present position, which causes the random walk's step size to be high. This percentage gradually rises as the search goes on in order to reduce the gap between the ideal position and the present position. As a result, exploration and exploitation are balanced appropriately.

The flag to change the direction of motion is denoted as F

$$F = \begin{cases} +1 \; if \; P \le 0.5 \\ -1 \; if \; P > 0.5 \end{cases} \tag{20}$$

Where, $P = 2 \times rand - C_4$

Step 8: Evaluation

Use objective function f to evaluate each object, and keep in mind the most successful answer so far.

Search and Rescue Optimization Algorithm

The mathematical model of the suggested approach for resolving a maximisation problem is discussed in this section [35]. In SAR, the positions of the people represent the optimization problem's solutions, and the quantity of hints discovered in these spots serves as the optimization problem for these solutions.

Step 1: Initialization

The optimisation process in AOA begins with the possible solutions shown in equation (20). The optimal candidate solution for each duplicate is deemed to be the most comprehensive or optimal solution, and they are created at random."

Step 2: Fitness function

Throughout the search, the group members collect clue information. When they discovered better clues in other locations, they left some clues behind, but information about them is used to further search efforts. The coordinates of the left clues are kept in the model we proposed memory matrix (matrix M), while the locations of the persons are kept in a location matrix (matrix X). The matrix M has the same dimensions as the matrix X. They are N D matrices, where N is the number of people and D is the size of the problem. The places of the discovered clues are contained in the clue matrix (matrix C). Two matrices, X and M, make up this matrix. Equation (21) demonstrates how to make C.

$$C=\begin{bmatrix} X \\ M \end{bmatrix}=\begin{bmatrix} X_{11} & \cdots & X_{1D} \\ \vdots & \ddots & \vdots \\ X_{N1} & \cdots & X_{ND} \\ M_{11} & \cdots & M_{1D} \\ \vdots & \ddots & \vdots \\ M_{N1} & \cdots & M_{ND} \end{bmatrix} \tag{21}$$

where X_{N1} represents the location of the first dimension for the N^{th} human, and M and X are the memory and human position matrices, respectively. M_{1D} is also where the D^{th} dimension for the first memory is located. The "social phase" and "individual phase," two stages of human search, are modelled as follows.

Step 3: Social Phase

The search direction is determined using the following equation by taking into consideration the justifications provided in the preceding section and accounting for a random clue among the discovered clues.

$$SD_i=\left(X_i-C_k\right),\ k\neq i, \tag{22}$$

where, X_i , C_k, and SD_i stand for the locations of the i^{th} human, the k^{th} clue, as well as the i^{th} human's search direction, respectively. A random integer value between 1 and $2N$ called k is selected such that $k\neq i$.

It is crucial to note that humans often search in a way that covers all targeted areas and avoids returning to previously examined areas. As a result, the search should be conducted in a way that limits group members' ability to move toward one another. To do this, travelling in the path of equation should not alter any of X_i dimensions (22). The binomial transformation function has been used to apply this limitation.

$$X_{i,j}^{'}=\begin{cases}\begin{cases}C_{k,j}+r1\times\left(X_{i,j}-C_{k,j}\right), & if\ f(C_k)>f(X_i),\\ X_{i,j}+r1\times\left(X_{i,j}-C_{k,j}\right), & otherwise\end{cases} & if\ r2<SE\ or\ j=j_{rand},\\ X_{i,j} & otherwise\end{cases} \quad (j=1,.....D), \tag{23}$$

where $X_{i,j}$ is denoted as the new location of the j^{th} dimension for the i^{th} human; $C_{k,j}$ is denoted as the location of the j^{th} dimension for the k^{th} found clue; $f(C_k)$ and $f(X_i)$ are the objective function values for the solutions C_k and X_i, respectively.

Step 4: Individual Phase

People conduct searches around their current location in the single phase using the social phase's concept of connecting various cues. In contrast to the social phase, the individual phase sees changes in every aspect of X_i . The following equation gives the new location of the i^{th} individual.

$$X_i^{'}=X_i+r3\times(C_k-C_m),\ i\neq k\neq m, \tag{24}$$

Where, k and m are denoted as the random integer numbers ranging between 1 and 2N. To prevent movement along with other clues k and m are chosen in such a way that $i\neq k\neq m$. $r3$ is denoted as a random number with a uniform distribution ranging among 0 and 1

Step 5: Boundary Control.

All solutions in metaheuristic algorithms must be found in the solution space; if they are outside the permitted solution space, they must be adjusted. Therefore, the following equation is used to alter the new position of a human if it is outside of the solution space.

$$X_{i,j}^{'}=\begin{cases}\dfrac{\left(X_{i,j}+X_j^{\max}\right)}{2}, & if\ X_{i,j}^{'}>X_j^{\max}\\ & (j=1,....,D)\\ \dfrac{\left(X_{i,j}+X_j^{\min}\right)}{2}, & if\ X_{i,j}^{'}>X_j^{\min}\end{cases} \tag{25}$$

Where, $X_j^{\max}$ and $X_j^{\min}$ are denoted as the values of the maximum and minimum threshold for j^{th} dimension

Step 6: Updating Information and Positions.

The group members are search in accordance with these two phases for each iteration, and following each phase, if the objective function value in location $X_i^{'}\left(f(X_i^{'})\right)$ is higher than the preceding one $\left(f(X_i)\right)$, the experience (X_i) are stored in a random location of the memory matrix (M) using equation (27) and this location are recognised as a new role using equation (28). The storage is not updated if this location is left, though.

$$Mn=\begin{cases}X_i, & IF\ f(X_i^{'})>f(X_i),\\ M_n, & otherwise\end{cases} \tag{27}$$

$$X_i=\begin{cases}X_i^{'}, & IF\ f(X_i^{'})>f(X_i),\\ X_i, & otherwise\end{cases} \tag{28}$$

where n is a random integer value between 1 and N, and M_n is the location of the nth recorded clue in the memory matrix. By using this kind of memory update, the algorithm becomes more diverse and is also better able to locate the global optimum.

Step 7: Abandoning Clues.

Time is an important component in search and rescue operations because injured lost persons may not survive if search and rescue crews take too long. Therefore, these procedures must be carried out in a fashion that allows for the fastest possible search of the largest space. Therefore, after a set number of searches in the area surrounding his or her current position, a humanmove to a new position if they are unable to locate any better hints. Unsuccessful search number (USN) is initially set to 0 for each individual to simulate this behaviour. The USN is set to 0 whenever a human discovers superior clues during the first or second phase of the search; otherwise, it isrise by 1 point as shown in the following equation.

$$USN = \begin{cases} USN_i + 1, & IF\ f(X_i^{'}) < f(X_i) \\ 0, & otherwise \end{cases} \tag{29}$$

where USN_i is the amount of times the person i has failed to uncover further useful information, When a person's USN exceeds the maximum number of unsuccessful searches (MU), they move to a random location in the search space using equation (30)

$$X_{i,j} = X_j^{\min} + r4 \times (X_j^{\max} - X_j^{\min}), \quad j = 1,....D, \tag{30}$$

where $r4$ is denoted as a random number with a uniform distribution ranging between 0 and 1.

Step 8: Control Parameters of SAR

SE (social effect) and MU are the two control factors for SAR (maximum unsuccessful search number). In the social phase, the SE is employed to regulate how group members interact with one another. This variable has a range of [0, 1]. Greater SE values boost convergence rate while also reducing algorithms' capacity for global search. The maximum number of unsuccessful searches before leaving a hint is indicated by the MU parameter. It falls between $\left[0,\ 2 \times T_{\max}\right]$ where $T_{\max}$ is the maximum number of iterations and $2 \times T_{\max}$ is the maximum number of searches that may be conducted by a single human. Humans are never abandon the hints for the larger ideals of the MU. On the one hand, low values of this variable force the third member of the group to finish the current clue's search before continuing to other locations. Magnitude of this parameter, on the other hand, result in a decrease in the likelihood of searching in other areas and an increase in searches focused on a single clue. The problem's dimension is directly related to the MU. The largest number of unsuccessful searches also rises as the search space does.

For each of the tests that followed, the SE value was set to 0.05, and the MU value was calculated using equation (31). These ratios for the SE and MU are suitable for addressing single-objective continuous optimization problems, according to the analysis of SAR parameters.

$$MU = 70 \times D \tag{31}$$

Result and Discussion

First case study: flat terrain

The proposed algorithm for generating optimal layouts for flat terrain was used to create 1,023 scenarios that explored all possible combinations of potential outlets. The results were plotted on a Pareto front, which shows the trade-off between two objectives: maximizing the ARI (a measure of the efficiency of the layout) and minimizing the LAI (a measure of the length of the pipes in the layout). From the Pareto front, five layouts were selected to demonstrate the potential consequences of different layout configurations. These layouts were chosen to have different numbers of outlets, or "DCs," which refers to the points where water is discharged from the system. The results showed that decentralization (using more outlets) generally led to lower LAI and higher ARI, resulting in lower overall costs for construction. This is because decentralized layouts use more main branches to transport water, rather than relying on a few main collectors as in centralized layouts. The NSI

layout function was not considered for the flat terrain case study, as it was assumed that there were no significant slopes in the ground.

	Total number of nodes	Total number of pipes	Total number of Subcatchments	Total lengths (m)	Average pipe lengths (m)	Total area (ha)	Total runoff area (ha)	Total number of outlet candidates	Elevation (m)
Flat areas	345	535	190	75500	68.78	520	390	11	19
Sttep areas	720	920	720	51217	56.89	190	120	12	590-560

The characteristics of the five selected layouts are summarized in Table 2. Layout number 3, which used 69% of the available outlets (7 out of 10), was found to be the optimal design based on the lowest LCC (total cost). The locations of the outlets for this layout are shown in Fig. 6. The results of this case study were compared with a similar study from the literature (also shown in Table 2). The LCC of the optimal design from this study was only 2% different from the optimal solution in the literature, but was obtained using fewer layout generations (1,023 versus 100,000). This demonstrates the efficiency of the proposed framework.

Design	LCC (M. Rials)	DC (%)	Average diameter (m/ m)	Average buried depth (m)	Maximum diameter (m)	LAI [M2]	ARI [-]
1(Fully centralized)	342625	100	0.66	2.27	2.0	122366	97.26
3	255685	68	0.53	1.93	1.5	97085	98.85
5	259760	66	0.54	1.92	1.5	101520	98.96

Resilience assessment

Resilience was assessed for the selected layouts, and the results are summarized in Table 3. Resilience in the context of sewer layouts can be characterized by two main factors: the total capacity of the system and the distribution of water flow. The centralized layout (number 1) had a larger storage capacity, but did not perform as well as the decentralized layouts during heavy storms due to its distribution of water flow. In comparison, the decentralized layouts had better flow distribution, which improved their resilience during heavy storms. The optimal decentralized solution in this study also outperformed the optimal centralized solution from the literature during all storm events, even though the latter had a larger total capacity. This is because the algorithm used in this study was able to achieve a better distribution of water flow.

Advantages and limitations

The proposed framework for generating optimal sewer layouts in this study is based on a deterministic combinatorial optimization approach, which reduces the computational demands compared to stochastic optimization methods such as metaheuristic algorithms. This allows the framework to handle larger network sizes and find near-optimal solutions more efficiently. However, the number of potential outlet combinations increases exponentially with the number of outlets, which may make the algorithm less efficient for very large networks with many scattered population patterns. In practice, the number of outlets is often limited by technical and operational constraints, and pre-processing can be used to reduce the number of outlet candidates. The results of the case studies in this study suggest that decentralization can lead to lower construction costs and better flow distribution in flat areas, but the opposite may be true in steep terrains with unfavorable slopes. Further research can be conducted to investigate the trade-offs between resilience and economic factors, as well as the sensitivity of the results to changes in the layout objectives.

Conclusions

This study presents multi-objective urban drainage networks based on graph theory and the ASROA technique. In order to enumerate layouts in the urban drainage networks, a deterministic multi-objective combinatorial optimization is coupled with a graph-theory-based layout generator, which is determined using the suggested ASROA technique. The ASROA technique solves the optimization challenge. Based on graph theory, the proposed deterministic framework may rapidly and efficiently develop decentralized and centralised sewage infrastructures that are almost optimal and feasible for both level and mountainous terrains.The provided layout costs may help pre-screen a number of layout solutions in flat and steep locations and help remove a large number of solutions before forwarding the solutions to the hydraulic design optimization.The framework may be used to any degree of input detail or to uneven topographies to provide the optimal design for mixed and separate sewage flows. Resilience analysis reveals that, even when the overall sewer capacity (to receive water flows) decreases, flow dispersion via several main collectors—achieved through decentralization—significantly lowers the amount of floodwaters, especially during intense storm events. We are certain that the community will be able to go on investigating and putting into practice decentralized solutions if our thorough work is integrated into mainstream UDN design software, which is used by both the commercial and non-commercial sectors, like the SWMM family.

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