Minimax Theory and its Applications

Volume 11 (2026), No. 5s, 292-306

Computing Performance Measures Of Bulk Arrival – Priority Queueing Model Using Different Defuzzification Techniques

G. Somasundara Ori[1], Andb.Abirami[2]

[1,2] Dwaraka Doss Goverdhan Doss Vaishnav College,Chennai-600106
Email:soamsundaraori@gmail.com, abirami.balagurunathan@gmail.com

Abstract
This paper presents a comparison study for the membership function of the system performance of bulk arrival priority queueing theory using different defuzzification techniques. Using two priority classes, we extend the membership functions where the customer service rate, batch-arrival rate, and other system parameters are all fuzzy values. Through Mathematical Programming Problem approach and the Conventional method, the fuzzy queues are reduced to a family of crisp queues based on Zadeh's extension notion using the α-cut strategy.
Furthermore, a comparison was made between the Crisp Values of the queues' system performance and Yager's methodology, the centroid method, and the First of Maximum (FOM) defuzzification techniques. This might be the case because, as stated in the literature, the features of the various defuzzification strategies match the application. A numerical problem is successfully addressed to demonstrate the applicability of several defuzzification methods. Triangular fuzzy numbers are used to demonstrate the validity of the idea.

Keywords: Membership Function, Fuzzy Number, performance measure of bulk arrival priority queue, ranking methods.

1.Introduction

The word fuzzy frequently denotes the word vague in the literature on fuzzy sets. It is helpful to discuss the connections between fuzziness and vagueness. Fuzzy sets seem to have always existed in the 20th century, based on the history of the sciences. In fact, a problem that surfaced in the first half of the century, mostly in philosophical circles, has been crystallized and made public by Zadeh's works [27]. Introduced by Zadeh, fuzzy sets are all about representing and manipulating data and information contents in the context of nonstatistical uncertainty. Fuzzy numbers provide a more realistic representation of the physical world than single-valued numbers by Stephen Dinagar et al. [23].

Systems based on fuzzy rules assess language rules through the use of composition, inference, and fuzzification techniques were given by G. Klir J. and B. Yuan [13]. Usually, fuzzy performance measure yields vague results that need to be transformed into clear output. Defuzzification is done to turn the fuzzy results into crisp ones.

Defuzzification, the final stage of approximation reasoning, involves substituting an appropriate crisp value for a fuzzy set.Eugene Roventaa and Tiberiu Spircuwere investigated whether averaging approaches may coexist with order and algebraic structures[20]. The most popular defuzzification methods into several groups and assess each group's prototypes based on the defuzzification standards are discussed by Werner Van Leekwijck and Etienne E. Kerre[25].

A weight calculation technique is used to estimate the mode of the fuzzy number, and the results are applied to a group decision-making exercise that takes into account numerous assessment criteria. CHI-BIN CHENG [5].

The output ofmembership functions with trapezoidal or triangular shapes, the technique can be extended to more broad shapes without significantly raising the on-line computational defuzzification.D.P. Madauet al. [14].The results of a traditional statistical study were compared by N. Mogharrebanand L. F. DiLalla with the output of the defuzzification techniques of Centroid, Bisector, Mean of Maximum (MOM), and Largest of Maximum (LOM).[15] H. Rouhparvar and A. Panahi are used a centroid-based distance approach, fuzzy numbers, both normal and non-normal, are compared and ranked based on their Euclidean distances from their centroid locations to the origin [21]. The simplified centroid equations developed for computational support to the two ranking systems' applications come from the field of analytical geometry by Ying-Ming Wanget. al [26].

Norman T. J. Baileyused the embedded Markov chain method to study the equilibrium distribution of queue length, and ergodicity is established when the average demand is smaller than the average supply available. [2].The fundamental idea is to use the Robust ranking technique to transform thefuzzy inter arrival rate, service rate, and batch sizes into crisp values by the traditional formulas for measuring queue performance suggested by B.Palpandi and G.Geetharamani [18]. After serving what considers to be less than "a" customer, the server takes a singleVacation. The new idea single-server finite capacity queue in which arrivals are controlledby the Markovian arrival process (MAP) and services are provided in batches of maximum size "b" with a minimum threshold "a"was proposed byU. C. Gupta and Karabisikdar[10].

Nagoor Gani and V. Ashok Kumarare calculated the upper and lower bounds of the system performance measure atthe possibility level, two mixed integer nonlinear programs are created.These applications apply the notion of Zadeh's extension. The performance measures fully retain the fuzziness of input data when some notions of bulk-arrival queuing systems are not evident since they are expressed by membership functions rather than by exact values. [16].

A multi-server batch-service queueing model in was studied and the process continues until the number of customers reaches a threshold. I. Adan, J. Resing were considered the largest batch size that can be used is ‘b’. It is demonstrated for this process that a limited sum of geometric expressions can be used to define the equilibrium probability for states in which all servers are active. [1].M. Chaudhry et al, explained that as long as customers are waiting in line, the server will start serving them right away; if not, they must wait for a customer to arrive before beginning service [3].

Hideaki Takagi mentioned in his article, a standard priority queue (without vacations) and two types of vacation models. The server waits for the first group of patrons to come if none have arrived yet. Under the multiple vacation approach, the server takes another vacation right away if it returns from one and discovers that the system is still empty. It keeps doing this until it discovers that there is always at least one customer waiting.[24]

Marcel.F.Neuts, used imbedded semi-Markov process to obtain the Characteristicsof the bulk serving or Customers are served in groups and he explained that afterthe serviced customers departure, there are fewer than Lcustomers present, the server has to wait until L customers are present before all one can begin serving. All customers present are serviced together if there are L or more but nearly K are still waiting ($L \leq K$). When K customers are waiting, a group of K customers enters service and the remaining customers are kept waiting. [17]

Customers arrive at random form a single queue then served in batches, the size of the batch either a fixed number or size of the queue, whichever is the smaller. The expressions are obtained for mean and variance of the waiting line by Downton, F [7] and same are obtained by Bailey used ‘imbedded Markov Chain’ technique devised by Kendall. N.K. Jaiswal noted that the same problem had been addressed using another technique.[2].

In the field of bulk service queueing models with multi-server, bulk service queue with priority queues and bulk service queue with working vacationsare developedfor good approximations by S. Sasikala and K.Indhira [22].The fuzzy batch arrival $FM^b/FM/I$ reduced to crisp M/M/1 priority service thePerformance Measures of the System were derived by Sushil Ghimire et al, and we used the same for our numerical calculations [9].Parametric programming to the analysis of fuzzy queues is for Four fuzzyqueues M/F/1, F/M/1, F/F/1 and FM/ FM/1 are exemplified by Chiang Kao et al. [12] In the broader context of service network modelling, Sundareswari [25] investigated nonlinear information-flow dynamics in distributed academic and library service networks, providing a related perspective on the mathematical analysis of distributed service systems.

J. Devaraj and D. Jayalakshmi, applies fuzzy set theory to examine the priority class queuing modelusing triangular fuzzy numbers suggest using fuzzy numbers to represent the arrival and service rates of two priority classes in Mathematical Programming technique to create the membership function of the system performance. The fuzzy queues are reduced to a family of crisp queues using the α-cut strategy and Zadeh's extension principle[6].

The system performances are totransform into crisp one by defuzzification. For triangular fuzzy numbers (TFN), different defuzzification techniques are applied. A few defuzzification techniques are examined numerically by Pinki[19].
H. Rouhparvar et al, using the incentre point of a triangle where the three bisector lines of its angles meet. Coordinates of incentre point were calculatedand solve problems of defuzzification and ranking fuzzy numbers. Finding the centroid position, by the distance is measured from the original point to the centroid point (Xo, Y0) [21].

In this paper we applied the conventional method and Mathematical programming problem method to find the interval of performance measure values using α-cut method.

The triangular fuzzy number is considered and calculated the intervals. Then using different defuzzification methods likeYager's method, Centroid method and First of Maximum method we find the performance measures and these values to be compared with crisp values.

Finally, the all defuzzied values are checked with the intervals of conventional method and Mathematical programming problem. Are all the values being inside the interval or not? And the interval range is wide enough or not? If the interval is wide enough and contains all defuzzied values then we conclude that the above method is better method to estimate the solution interval through α-cut method.

This paper is coordinated as follows: second segment gives some basic definitions of this research work. In third section construct a mathematical model offuzzy batch priority queueing model performance measurement and list its formula. Fourth section with numerical example to calculate the values by different defuzzification methods continuedwith discussions in fifth segment and sixth segment concludes the paper.

2.Preliminaries

In this section we recall some basic definition.

Definition 2.1. A fuzzy set is a set where the members are allowed to have partialmembership and hence the degree of membership varies from 0 to 1. It is expressed as$\tilde{A}$={(x,$\mu_{\tilde{A}}(X)$)/X∈Z}where Z is the universe of discourse and $\mu_{\tilde{A}}(X)$isarealnumber, $\mu_{\tilde{A}}(X)$=0 or 1,ie,x is a non-member in $\tilde{A}$ if $\mu_{\tilde{A}}(X)$=0 and X is a member in $\tilde{A}$if$\mu_{\tilde{A}}(X)$=1.

Definition 2.2 If a fuzzy set $\tilde{A}$is defined on X , for any α ∈ [0.1]], theα-cuts of thefuzzyset $\tilde{A}$isrepresented by $\tilde{A}_{\alpha} = \{X/\mu_{\tilde{A}}(X) \geq \alpha, X \in Z\} = \{l_{\tilde{A}}(\alpha), u_{\tilde{A}}(\alpha)\}$,where $l_{\tilde{A}}(\alpha)$and$u_{\tilde{A}}(\alpha)$represent the lower bound and upper bound of the α-cutof$\tilde{A}$respectively.

Definition2.3. The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set $\tilde{A}_{\alpha} = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ where α ∈ [0,1].

Definition2.4. The support of a fuzzy set A is the crisp set such that it is represented as sup$\tilde{A}$(X) = $\{x \in X/\mu_{\tilde{A}}(x) > 0\}$. Thus, support of a fuzzy set is the set of all memberswithastrongα-cut,where α=0.

Definition2.5. Triangular fuzzy number

A fuzzy number $\tilde{A} = (a, b, c)$ is called triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c. \end{cases}$$

Definition2.6. Conversion of triangular fuzzy number into Interval using α − cut

Let $\tilde{A} = \{a, b, c\}$ be the triangular fuzzy number then to fine α −cut of $\tilde{A}$. we first set α equal to the left and right membership function of $\tilde{A}$.

That is α $= \frac{x-a}{b-a}$ and α $= \frac{c-x}{c-b}$.Expressing x in terms of α we have,

x = α$(b-a)+a$ and x = −α$(c-b)+c$.

Therefore we can write the fuzzy interval in terms of α − cut interval:

$\tilde{A}_{\alpha} = [\alpha(b-a)+a, -\alpha(c-b)+c]$.

Definition 2.7. conventional method

In the conventional Method $l_{p(\alpha)}$ and $u_{p(\alpha)}$are represent as,

$$l_{p(\alpha)} = min\ p(a, s)$$

Such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}, i = 1,2$

$$l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$$

and

$$u_{p(\alpha)} = max\ p(a,s)$$

Such that $l_{A_i(\alpha)} \leq a \leq u_{A_i(\alpha)}, i = 1,2$

$$l_{s(\alpha)} \leq s \leq u_{s(\alpha)}$$

The $l_{W_{q1}(\alpha)}$ is found when r1 , r2 and k approaches their lower bounds.The $u_{W_{q1}(\alpha)}$ is found when r1 , r2 and k approaches their upper bounds. Apply this on the performance measures of respective models.

Definition 2.8. Mathematical programming problem model

In the Mathematical programming approach $l_{p(\alpha)}$ and $u_{p(\alpha)}$are represent as,

$$l_{p(\alpha)} = min\ p(a,s) \ldots\ldots\ldots\

Find the required values.

Definition2.11. First Of Maximum Model

If given as fuzzy parameters (triangular fuzzy numbers) then they can be represented as respectively$\tilde{A} = (a_1, a_2, a_3)$ thenparameter will become:

$FM(\tilde{A}) = FM(a_1, a_2, a_3) = a_1 \ or \ a_2 \ or \ a_3$ by first attain the full height of membership will be assigned and that value will be used to measure the other required performance measures.

Definition 2.12 CRISP VALUE

If given as fuzzy parameters (triangular fuzzy numbers) then they can be represented as respectively$\tilde{A} = (a_1, a_2, a_3)$ thenparameter will become:

$$Cr(\tilde{A}) = \frac{1}{2}(U_{a_i} + L_{a_i})$$

After that we use this $Cr(\tilde{A})$ values in the performance measure and

Find the required values.

Development of model for Fuzzy Bulk Queue Arrival with Priority ModelFM^b/FM/I solving by Mathematical programming approach

We take into consideration the fuzzy queueing model FM^b/FM/1 of bulk arrival rate size 'b' with priority discipline to illustrate and compare the results of defuzzied values.

Construction Of Model

We consider the FM^b/FM/1 fuzzy batch priority queuing model. Every arrival of customer is classified as belonging to any one of two priority types. Then, we assumed that, in batches of size "b," arrivals of type 1 or higher priority have a mean arrival rate of $\widetilde{\lambda_1}$ and those of type 2 or lower priority or no priority have a mean arrival rate of $\widetilde{\lambda_2}$, so that $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}$.is true. Customers of type 1 are allowed to get service prior to all other customers without pre-emption. We presumptively used a single server system and an infinite number of calling sources.

The average system lengths of type 1 and type 2 priority queues are represented in this model by the letters $\bar{L}_1$and $\bar{L}_2$, respectively. The letters $\bar{L}_{q1}$and $\bar{L}_{q2}$ stand for the length of the queues of type 1 and type 2 priority queues, respectively. The average waiting times for customers in the type 1 and type 2 priority queues, respectively, are denoted by the symbols $\bar{W}_{q1}$and $\bar{W}_{q2}$, respectively. Devaraj & Jayalakshmi [23] explored priority queues using a fuzzy approach; both the system capacity and the population of calling sources are infinite.

In this model, we considered that each batch's size is 'b' and that the batch arrival rate is finite. All service times are distributed according to an exponential distribution with fuzzy service rate $\tilde{\mu}$at a single-server facility in batches as a Poisson process with group arrival mean rate$\tilde{\lambda}$, where $\tilde{\lambda}$, is a fuzzy number. The batch size 'b' of arrival is represented by triangular fuzzy number. The size of the calling population is infinite, and the system capacity is infinite. Customers are served using a priority model. The arrival rate triangular fuzzy number is represented by various intervals of confidence using α- cuts.

Representation Of Fuzzy Sets

The inter-arrival times $\tilde{A}_i$, i = 1,2of units in the type 1 and type 2 priority and service times $\tilde{S}$are represented by the fuzzy sets in the following form in the single server queuing system $FM^b/FM/1$

$$\tilde{A}_i = \{(a, \mu_{\widetilde{A_i}}(a))/a \in X\}, i = 1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\tilde{S} = \{(s, \mu_{\tilde{s}}(s))/s \in Y\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where X is universal crisp set of the inter arrival rate with its membership function $\mu_{\widetilde{A_i}}(a), i = 1,2$and Y is also crisp universal set of service time with its membership function $\mu_{\tilde{s}}(s)$.

The fuzzy batch arrival FM^b/FM/I reduced to crisp M/M/1priority service

Using the concept of $\alpha-$ cut the $FM^b/FM/1$ queue with priority can be in the following form by [9]

$L_{q1} = \frac{\rho(b-1+2\frac{\lambda_1}{\mu})}{2\left(1-\frac{\lambda_1}{\mu}\right)}$…… (3)

$L_{q2} = \frac{\rho(b-1+2\frac{\lambda_2}{\mu})}{2(1-\rho)\left(1-\frac{\lambda_2}{\mu}\right)}$…… (4)

$W_{q1} = \frac{b+2\rho-1}{2\mu\left(1-\frac{\lambda_1}{\mu}\right)}$…… (5)

$W_{q2} = \frac{b+2\rho-1}{2(\mu-\lambda)\left(1-\frac{\lambda_2}{\mu}\right)}$……..…… (6)

Where $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$ are the arrival rates of type 1 priority and type 2 priority units respectively and $\tilde{\mu}$ is the service rate. Further $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}$ and

$\rho = \frac{\lambda b}{\mu}$ from [9].

If the functions $l_{p(\alpha)}$ and $u_{p(\alpha)}$ are not invertible with respect to α, the membership functions $\mu_{p\,(\tilde{A},\tilde{S})}(z)$ in not derived. But we can trace the graph of $\mu_{p(\tilde{A},\tilde{S})}(z)$ from the $\alpha-$ cuts of $[l_{p(\alpha)}, u_{p(\alpha)}]$.

This procedure can be applied to find the membership functions $[l_{s(\alpha)}, u_{s(\alpha)}]$ for the batch priority queuing model with priority can be obtained.

Comparison study for membership function using different Defuzzification Methods

Suppose the arrival rate of the two types of priority queues with the same service rate are fuzzy numbers represented by$\tilde{A}_1 = [2,4,5], \tilde{A}_2 = [3,5,6]$ and

$\tilde{S} = [35,65,80]$ per hour respectively with batch size b=3. The $\alpha-$cut of the membership functions $u_{\tilde{A}_1}(\alpha), u_{\tilde{A}_2}(\alpha)$ and $u_{\tilde{S}}(\alpha)$ are $[2\alpha+2, 5-\alpha]$,

$[2\alpha+3, 6-\alpha]$ and $[30\alpha+35, 80-15\alpha]$ respectively using definition 2.6. From equation (3), (4), (5) and (6) are formulated to derive the membership function

$\bar{L}_{q1}, \bar{L}_{q2}, \bar{W}_{q1}$and $\bar{W}_{q2}$.

The performance functions of

(i) $\bar{L}_{q1}$ - average queue length of highest priority type 1

(ii) $\bar{L}_{q2}$ - average queue length of less priority type 2

(iii) $\bar{W}_{q1}$- average waiting time of units' highest priority in the queue type 1

(iv) $\bar{W}_{q2}$- average waiting time of units less priority in the queue type 2

Performance Measures

The performance measures $\bar{L}_{q1}$, $\bar{L}_{q2}$, $\bar{W}_{q1}$ and $\bar{W}_{q2}$ are differ only in their objective functions.

Conventional Method Based On α-Cut Approach

For type 1 or higher priority- Length of the queue

The objective function: The performance measure of $\bar{L}_{q1}$

$$\left.\begin{aligned} l_{L_{q1}(\alpha)} &= \min\left\{\frac{b\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\frac{r1}{k}\right)}\right\} \\ u_{L_{q1}(\alpha)} &= max\left\{\frac{b\left(\frac{r1+r2}{k}\right)\left(b-1+2\left(\frac{r1}{k}\right)\right)}{2\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that \quad & 2+2\alpha \le r1 \le 5-\alpha \\ & 3+2\alpha \le r2 \le 6-\alpha \\ & 35+30\alpha \le k \le 80-15\alpha \end{aligned}\right\}$$

Where $0 < \alpha \le 1$.

$l_{L_{q1}(\alpha)}$ is found when r1, r2 and k approaches their lower bounds.

Consequently, the optimal solution is

$$l_{L_{q1}(\alpha)} = \frac{555+924\alpha+384\alpha^2}{1155+1970\alpha+840\alpha^2} \ldots\ldots\ldots\

At $\alpha = 0.5$, then $u_{L_{q2}(0.5)} = 0.821773485$

For type 1priority - waiting time in the queue

The performance functions of $\widetilde{W_{qP1}}$and $\widetilde{W_{qP2}}$ are derived from the respective parametric programs.

$$\left.\begin{aligned} l_{W_{q1}(\alpha)} &= min\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ u_{W_{q1}(\alpha)} &= max\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that\quad & 2+2\alpha \le r1 \le 5-\alpha \\ & 3+2\alpha \le r2 \le 6-\alpha \\ & 35+30\alpha \le k \le 80-15\alpha \end{aligned}\right\}$$

Where $0 < \alpha \le 1$.

$l_{W_{q1}(\alpha)}$ is found when r1, r2 and k approaches their lower bounds

Consequently, the optimal solution is

$$l_{W_{q1}(\alpha)} = \frac{50+42\alpha}{1155+1970\alpha+840\alpha^2} \quad \text{..... (11)}$$

At $\alpha = 0.5$, then $l_{W_{q1}(0.5)} = 0.030212765$

Also, $u_{W_{q1}(\alpha)}$ is found when r1, r2 and k approaches their upper bounds.

Consequently, the optimal solution is

$$u_{W_{q1}(\alpha)} = \frac{113-21\alpha}{6000-2245\alpha+210\alpha^2} \quad \text{..... (12)}$$

At $\alpha = 0.5$, then $u_{L_{q1}(0.5)} = 0.020791075$

Fortype 2 priority - waiting time in the queue

$$\left.\begin{aligned} l_{W_{q2}(\alpha)} &= min\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)}\right\} \\ u_{W_{q2}(\alpha)} &= max\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)}\right\} \\ such\ that\quad & 2+2\alpha \le r1 \le 5-\alpha \\ & 3+2\alpha \le r2 \le 6-\alpha \\ & 35+30\alpha \le k \le 80-15\alpha \end{aligned}\right\}$$

Where $0 < \alpha \le 1$.

$l_{W_{q2}(\alpha)}$ is found when r1, r2 and k approaches their lower bounds.

Then, the optimal solution is

$$l_{W_{q2}(\alpha)} = \frac{50+42\alpha}{960+1672\alpha+728\alpha^2} \quad \text{..... (13)}$$

At $\alpha = 0.5$, then $l_{w_{q2}(0.5)} = 0.035894843$

Also, $u_{W_{q2}(\alpha)}$ is found when r1, r2 and k approaches their upper bounds.

Consequently, the optimal solution is

$$u_{W_{q2}(\alpha)} = \frac{113-21\alpha}{5106-1928\alpha+182\alpha^2} \ldots\ldots\ldots\

Where $0 < \alpha \leq 1$.

$l_{L_{q2}(\alpha)}$ is found when r1 and r2 approaches their lower bounds and k approaches its upper bound. Then, the optimal solution is

$$l_{L_{q2}(\alpha)} = \frac{1245+801\alpha-156\alpha^2}{5005-3184\alpha+459\alpha^2} \quad \text{..... (21)}$$

At $\alpha = 0.5$, then $l_{L_{q2}(0.5)} = 0.455389412$

Also, $u_{L_{q2}(\alpha)}$ is found when r1 and r2 approaches their upper bounds and k approaches its lower bound.

Consequently, the optimal solutions for (15) is

$$u_{L_{q2}(\alpha)} = \frac{1353+711\alpha-174\alpha^2}{58+1106\alpha+1116\alpha^2} \quad \text{..... (22)}$$

At $\alpha = 0.5$, then $u_{L_{q2}(0.5)} = 1.870786517$

For type 1and type 2 priority - waiting time in the queue

The performance functions of $\widetilde{W_{qP1}}$and $\widetilde{W_{qP2}}$ are derived from the respective parametric programs.

$$\left.\begin{array}{rl} l_{W_{q1}(\alpha)} & = min\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that & 2+2\alpha \leq r1 \leq 5-\alpha \\ & 3+2\alpha \leq r2 \leq 6-\alpha \\ & 35+30\alpha \leq k \leq 80-15\alpha \end{array}\right\} \quad \text{..... (23)}$$

and

$$\left.\begin{array}{rl} u_{W_{q1}(\alpha)} & = max\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2k\left(1-\frac{r1}{k}\right)}\right\} \\ such\ that & 2+2\alpha \leq r1 \leq 5-\alpha \\ & 3+2\alpha \leq r2 \leq 6-\alpha \\ & 35+30\alpha \leq k \leq 80-15\alpha \end{array}\right\} \quad \text{..... (24)}$$

$$\left.\begin{array}{rl} l_{W_{q2}(\alpha)} & = min\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)}\right\} \\ such\ that & 2+2\alpha \leq r1 \leq 5-\alpha \\ & 3+2\alpha \leq r2 \leq 6-\alpha \\ & 35+30\alpha \leq k \leq 80-15\alpha \end{array}\right\} \quad \text{..... (25)}$$

and

$$\left.\begin{array}{rl} u_{W_{q2}(\alpha)} & = max\left\{\frac{\left(b+2\left(\frac{3(r1+r2)}{k}\right)-1\right)}{2(k-(r1+r2))\left(1-\frac{r2}{k}\right)}\right\} \\ such\ that & 2+2\alpha \leq r1 \leq 5-\alpha \\ & 3+2\alpha \leq r2 \leq 6-\alpha \\ & 35+30\alpha \leq k \leq 80-15\alpha \end{array}\right\} \quad \text{..... (26)}$$

For the above equations $l_{W_{q1}(\alpha)}$,$l_{W_{q2}(\alpha)}$ is found when r1 and r2 approaches their lower bounds and k approaches its upper bound. Also, $u_{W_{q1}(\alpha)}$,$u_{W_{q2}(\alpha)}$ is found when r1 and r2 approaches their upper bounds and k approaches its lower bound.

The optimal solutions for (23), (24) are

$$l_{W_{q1}(\alpha)} = \frac{95-3\alpha}{6240-2530\alpha+255\alpha^2} \quad \text{..... (27)}$$

At $\alpha = 0.5$, then $l_{W_{q1}(0.5)} = 0.018556189$

$$u_{W_{q1}(\alpha)} = \frac{68+24\alpha}{1050+1985\alpha+930\alpha^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(28)}$$

At $\alpha = 0.5$, then $u_{W_{q1}(0.5)} = 0.035164835$

The optimal solutions for (25), (26) are

$$l_{W_{q2}(\alpha)} = \frac{95-3\alpha}{5313-2174\alpha+221\alpha^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(29)}$$

At $\alpha = 0.5$, then $l_{W_{q2}(0.5)} = 0.021839416$

$$u_{W_{q2}(\alpha)} = \frac{68+24\alpha}{696+1672\alpha+992\alpha^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

At $\alpha = 0.5$, then $u_{W_{q2}(0.5)} = 0.04494382$

Yager's method

Given $\tilde{A}_1 = [2,4,5], \tilde{A}_2 = [3,5,6]$ and $\tilde{S} = [35,65,80]$ per hour respectively with batch size b=3.

$$R(\lambda_1) = R(2,4,5) = \left(a_{\lambda_1}^L, a_{\lambda_1}^U\right) = [2+2\alpha, 5-\alpha]$$

$$= \int_0^1 0.5(2+2\alpha+5-\alpha)\,d\alpha = \int_0^1 0.5(7+\alpha)\,d\alpha$$

$$= 0.5\left[7\alpha + \frac{\alpha^2}{2}\right]_0^1 = 3.75$$

$$R(\lambda_2) = R(3,5,6) = \left(a_{\lambda_2}^L, a_{\lambda_2}^U\right) = [3+2\alpha, 6-\alpha]$$

$$= 0.5\left[9\alpha + \frac{\alpha^2}{2}\right]_0^1 = 4.75$$

$$R(\lambda) = R(5,9,11) = \left(a_{\lambda}^L, a_{\lambda}^U\right) = [5+4\alpha, 11-2\alpha]$$

$$= 0.5[16+\alpha^2]_0^1 = 8.5$$

$$R(\mu) = R(35,65,80) = \left(a_{\mu}^L, a_{\mu}^U\right) = [35+30\alpha, 80-15\alpha]$$

$$= 0.5\left[115\alpha + \frac{15\alpha^2}{2}\right]_0^1 = 61.25$$

$\rho = \frac{b\lambda}{\mu}$ and $b = 3$.

$$L_{q_1} = \frac{\rho\left(b-1+2\frac{\lambda_1}{\mu}\right)}{2\left(1-\frac{\lambda_1}{\mu}\right)} = \frac{3\lambda(\mu+\lambda_1)}{\mu(\mu-\lambda_1)} = \frac{3(8.5)(61.25+3.75)}{61.25(61.25-3.75)} = 0.47062991$$

$$L_{q_2} = \frac{\rho\left(b-1+2\frac{\lambda_2}{\mu}\right)}{2(1-\rho)\left(1-\frac{\lambda_2}{\mu}\right)} = \frac{3\lambda(\mu+\lambda_2)}{(\mu-3\lambda)(\mu-\lambda_2)} = \frac{3(8.5)(61.25+4.75)}{(61.25-3(8.5))(61.25-4.75)} = 0.83321987$$

$$W_{q_1} = \frac{b+2\rho-1}{2\mu\left(1-\frac{\lambda_1}{\mu}\right)} = \frac{\mu+3\lambda}{\mu(\mu-\lambda_1)} = \frac{(61.25+3(8.5))}{61.25(61.25-3.75)} = 0.024631765$$

$$W_{q_2} = \frac{b+2\rho-1}{2(\mu-\lambda)\left(1-\frac{\lambda_2}{\mu}\right)} = \frac{\mu+3\lambda}{(\mu-\lambda)(\mu-\lambda_2)} = \frac{(61.25+3(8.5))}{(61.25-8.5)(61.25-4.75)} = 0.029107075$$

centroid Method

Given $\tilde{A}_1 = [2,4,5], \tilde{A}_2 = [3,5,6]$ and $\tilde{S} = [35,65,80]$ per hour respectively with batch size b=3.

$$Ç(\lambda_1) = Ç(2,4,5) = \frac{\lambda_1^1 + \lambda_1^2 + \lambda_1^3}{3} = \frac{1}{3}[2+4+5]$$

$$= \frac{1}{3}[11] = \frac{11}{3}$$

$$Ç(\lambda_2) = Ç(3,5,6) = \frac{\lambda_2^1 + \lambda_2^2 + \lambda_2^3}{3} = \frac{1}{3}[3+5+6] = \frac{14}{3}$$

$$Ç(\lambda) = Ç(5,9,11) = \frac{\lambda_{1+2}^1 + \lambda_{1+2}^2 + \lambda_{1+2}^3}{3} = \frac{1}{3}[5+9+11] = \frac{25}{3}$$

$$Ç(\mu_s) = Ç(35,65,80) = \frac{\mu_s^1 + \mu_s^2 + \mu_s^3}{3} = \frac{1}{3}[35+65+80] = \frac{180}{3}$$

$\rho = \frac{b\lambda}{\mu}$ and $b = 3$.

$$L_{q_1} = \frac{\rho\left(b - 1 + 2\frac{\lambda_1}{\mu}\right)}{2\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_1)}{\mu(\mu - \lambda_1)} = \frac{3\left(\frac{25}{3}\right)\left(\frac{180}{3} + \frac{11}{3}\right)}{\frac{180}{3}\left(\frac{180}{3} - \frac{11}{3}\right)} = 0.470907297$$

$$L_{q_2} = \frac{\rho\left(b - 1 + 2\frac{\lambda_2}{\mu}\right)}{2(1-\rho)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_2)}{(\mu - 3\lambda)(\mu - \lambda_2)} = \frac{3\left(\frac{25}{3}\right)\left(\frac{180}{3} + \frac{14}{3}\right)}{\left(\frac{180}{3} - 3\left(\frac{25}{3}\right)\right)\left(\frac{180}{3} - \frac{14}{3}\right)} = 0.834767642$$

$$W_{q_1} = \frac{b + 2\rho - 1}{2\mu\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{\mu + 3\lambda}{\mu(\mu - \lambda_1)} = \frac{\left(\frac{180}{3} + 3\left(\frac{25}{3}\right)\right)}{\frac{180}{3}\left(\frac{180}{3} - \frac{11}{3}\right)} = 0.025147928$$

$$W_{q_2} = \frac{b + 2\rho - 1}{2(\mu - \lambda)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{\mu + 3\lambda}{(\mu - \lambda)(\mu - \lambda_2)} = \frac{\left(\frac{180}{3} + 3\left(\frac{25}{3}\right)\right)}{\left(\frac{180}{3} - \frac{25}{3}\right)\left(\frac{180}{3} - \frac{14}{3}\right)} = 0.02973183$$

First of Maximum (FOM)

Given $\tilde{A}_1 = [2,4,5], \tilde{A}_2 = [3,5,6]$ and $\tilde{S} = [35,65,80]$ per hour respectively with batch size b=3.

$$FM(\lambda_1) = FM(2,4,5) = 4$$

$$FM(\lambda_2) = FM(3,5,6) = 5$$

$$FM(\lambda) = FM(5,9,11) = 9$$

$$FM(\mu) = FM(35,65,80) = 65$$

$\rho = \frac{b\lambda}{\mu}$ and $b = 3$.

$$L_{q_1} = \frac{\rho\left(b - 1 + 2\frac{\lambda_1}{\mu}\right)}{2\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_1)}{\mu(\mu - \lambda_1)} = \frac{3(9)(65+4)}{65(65-4)} = 0.469861286$$

$$L_{q_2} = \frac{\rho\left(b - 1 + 2\frac{\lambda_2}{\mu}\right)}{2(1-\rho)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_2)}{(\mu - 3\lambda)(\mu - \lambda_2)} = \frac{3(9)(65+5)}{(65-3(9))(65-5)} = 0.8289473$$

$$W_{q_1} = \frac{b + 2\rho - 1}{2\mu\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{\mu + 3\lambda}{\mu(\mu - \lambda_1)} = \frac{(65 + 3(9))}{65(65-4)} = 0.025147928$$

$$W_{q_2} = \frac{b + 2\rho - 1}{2(\mu - \lambda)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{\mu + 3\lambda}{(\mu - \lambda)(\mu - \lambda_2)} = \frac{(65 + 3(9))}{(65 - 9)(65 - 5)} = 0.02973183$$

Crisp Values

Given $\tilde{A}_1 = [2,4,5], \tilde{A}_2 = [3,5,6]$ and $\tilde{S} = [35,65,80]$ per hour respectively with batch size b=3.

$$Cr(\lambda_1) = \frac{1}{2}\left(U_{\lambda_1} + L_{\lambda_1}\right) = \frac{1}{2}(5 + 2) = 3.5$$

$$Cr(\lambda_2) = \frac{1}{2}\left(U_{\lambda_2} + L_{\lambda_2}\right) = \frac{1}{2}(6 + 3) = 4.5$$

$$\text{Cr}(\lambda) = \tfrac{1}{2}(U_\lambda + L_\lambda) = \tfrac{1}{2}(11 + 5) = 8$$

$$Cr(\mu) = \frac{1}{2}\left(U_\mu + L_\mu\right) = \frac{1}{2}(80 + 35) = 57.5$$

$\rho = \frac{b\lambda}{\mu}$ and $b = 3$.

$$L_{q_1} = \frac{\rho\left(b - 1 + 2\frac{\lambda_1}{\mu}\right)}{2\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_1)}{\mu(\mu - \lambda_1)} = \frac{3(8)(57.5 + 3.5)}{57.5(57.5 - 3.5)} = 0.471497584$$

$$L_{q_2} = \frac{\rho\left(b - 1 + 2\frac{\lambda_2}{\mu}\right)}{2(1 - \rho)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{3\lambda(\mu + \lambda_2)}{(\mu - 3\lambda)(\mu - \lambda_2)} = \frac{3(8)(57.5 + 4.5)}{(57.5 - 3(8))(57.5 - 4.5)} = 0.8289473$$

$$W_{q_1} = \frac{b + 2\rho - 1}{2\mu\left(1 - \frac{\lambda_1}{\mu}\right)} = \frac{\mu + 3\lambda}{\mu(\mu - \lambda_1)} = \frac{(57.5 + 3(8))}{57.5(57.5 - 3.5)} = 0.026247987$$

$$W_{q_2} = \frac{b + 2\rho - 1}{2(\mu - \lambda)\left(1 - \frac{\lambda_2}{\mu}\right)} = \frac{\mu + 3\lambda}{(\mu - \lambda)(\mu - \lambda_2)} = \frac{(57.5 + 3(8))}{(57.5 - 8)(57.5 - 4.5)} = 0.03106537$$

Discussion

Table 1. Finding Lower and upper limits of Performance measure of Conventional method and Mathematical Programming Problem

	Convention model	Mathematical Programming Problem
$l_{L_{q1}(0.5)}$	0.473617021	0.0314661374
$u_{L_{q1}(0.5)}$	0.468559837	0.7189281318
$l_{L_{q2}(0.5)}$	0.850074962	0.455389412
$u_{L_{q2}(0.5)}$	0.821773485	1.870786517
$l_{W_{q1}(0.5)}$	0.030212765	0.018556189
$u_{W_{q1}(0.5)}$	0.020791075	0.035164835
$l_{W_{q2}(0.5)}$	0.035894843	0.021839416
$u_{W_{q2}(0.5)}$	0.024477611	0.04494382

Table 2. For $FM^b/FM/1$ queueing performance measure by Yager's, Centroid, FOM and Crisp values.

	Yagers	Centroid	FOM	Crisp
L_{q1}	0.47062991	0.470907297	0469861286	0.471497584
L_{q2}	0.83321987	0.834767642	0.8289473	0.8289473
W_{q1}	0.024631765	0.025147928	0.025147928	0.026247987
W_{q2}	0.029107075	0.02973183	0.02973183	0.03106537

Table 2 shows that the $\boldsymbol{L_{q_1}}$ values for all four models are within and quite close to the intervals of the mathematical programming method. The crisp value is the highest among all model evaluations. Consequently, the computed $\boldsymbol{L_{q_1}}$priority queue values with significance difference are quite small.

All of the lower limit numbers in Table 1 of the Conventional model are higher than the upper limit value. Therefore, all of the limitations are contained within the bounds of the Mathematical Programming problem and are valid. This demonstrates that compared to other approaches, Mathematical Programming Problem limits have a greater range. The Mathematical Programming problem boundaries contain all of the methods performance measure values.

The differences for $\boldsymbol{L_{q_1}, L_{q_2}, W_{q_1}}and\boldsymbol{W_{q_2}}$ when compared to the Crisp value using Yager's technique are $4.404593\text{x}10^{-3}$, $-4.27257\text{x}10^{-3}$, $1.616216\text{x}10^{-3}$and $1.958295\text{x}10^{-3}$respectively. When Crisp values are compared using the Centroid method, we obtain $5.90287\text{x}10^{-4}$, $-5.820342\text{x}10^{-3}$, $1.100059\text{x}10^{-3}$ and $1.33354\text{x}10^{-3}$for $\boldsymbol{L_{q_1}, L_{q_2}, W_{q_1}}and\boldsymbol{W_{q_2}}$ respectively. When we compare Crisp values using the FOM method, we obtain $1.636298\text{x}10^{-3}$, 0, $1.100059\text{x}10^{-3}$ and $1.33354\text{x}10^{-3}$for $\boldsymbol{L_{q_1}, L_{q_2}, W_{q_1}}and\boldsymbol{W_{q_2}}$ respectively.This demonstrates that all methods have fairly similar values and do not differ much. However, the approach for solving Mathematical Programming Problem provides a broad interval of solutions, while all other approaches fall within this range exclusively. We therefore draw the conclusion that the large interval range of performance measure values is best provided by the Mathematical Programming Problem method.

This paper compares and calculates two approaches: the conventional method and the Mathematical Programming Problemmethod. The limit values obtained using this conventional method was not what is anticipated. Therefore, the way of solving Mathematical Programming Problem produces more accurate interval values. The remaining performance values are then all lies, correspondingly, between these intervals. The interval values of the mathematical programming problem's crisp values for $\boldsymbol{W_{q_1}}$and$\boldsymbol{W_{q_2}}$ are extremely near to their middle values.

Conclusion

The defuzzification literature has put forth a wide range of techniques. Comparative studies have not, however, kept pace with the number of methodologies, with the exception of a small number of in-depth investigations. In this study, we used numerical examples to compare various defuzzification techniques. The study's findings emphasized both the positive effects as well as limitations of various defuzzification techniques. In order to obtain the interval of performance measure values using the α-cut approach, we employed both the Conventional method and the Mathematical Programming Problem in this paper. Next, we computed the intervals using a triangular fuzzy number. Next, using Yager's approach, Centroid method, and First of Maximum method for defuzzification, the performance measures $\boldsymbol{L_{q_1}, L_{q_2}, W_{q_1}}and\boldsymbol{W_{q_2}}$ were computed and compared to crisp values.
Lastly, a Mathematical Programming Problem and the intervals of the conventional method are used to verify all of the defuzzied performance measure values. We discovered that the interval range is sufficiently large and that all of the values of the defuzzied performance measures fall inside the intervals. We determine that the aforementioned Mathematical Programming Problem provides a more accurate way to estimate the solution interval using the α-cut method. We believe that this work provides evidence for the superior interval of performance metrics that Mathematical Programming Problem can yield. By comparing the different models, we intend to spark a discussion on defuzzification techniques and lead to more research on the subject.

References

1. Adan.I, Resing.J.,(2000),“multi-server batch-service systems”, Statistica Neer-landica, 54, 202-220.
2. Bailey. N.T.J.,(1954),“On queuing process with bulk service”, Journal of the Royal Statistical Society,Vol.16,Pp.80-87.
3. Salman, B. B. (2026). New Four-Step Iteration in Convex Metric Spaces. *Results in Nonlinear Analysis*, *9*(2), 165-178.
4. Cheng.C.H., (1998),“A new approach for ranking fuzzy numbers by distance method*”,*Fuzzy Sets and Systems, Vol.95(3), Pp.307–317.DOI:10.1016/s0165-0114(96)00272-2.
5. Cheng. C.H.,(2004), “Group Opinion Aggregation Based on a Grading Process: A Method for Constructing Triangular Fuzzy Numbers”, Computers and Mathematics with Applications, Vol.48, Pp.1619-1632, DOI: 10.1016/j.camwa.2004.03.008.
6. Devaraj.J., Jayalakshmi.D.,(2012),“A Fuzzy approach to priority queues”, International Journal of Fuzzy Mathematics and Systems,Vol 2(4), Pp.479-488.
7. Downton. F*., (1955), “*Waiting Time in Bulk Service Queues*”,*Journal of the Royal Statistical Society: Series B (Methodological)*,*Vol.17(2), Pp. 256–261. *DOI:10.1111/j.2517-6161. 1955.tb00199. x.*
8. Dubois.D., Ostasiewicz.W. and Prade.H., (2000), “Fuzzy Sets: History and Basic Notions*”,*The Handbooks of Fuzzy Sets Series*,* Pp.21–124.
9. Ghimire. S., Ghimire. R. P.and Thapa.G.B., (2014), “Mathematical Models of M^b/M/1 Bulk Arrival Queueing System”, Journal of the Institute of Engineering, Vol.10(1), Pp.184-191, DOI:10.3126/jie.v10i1.10899.
10. Gupta.U., Sikdar.K., (2004), “A finite capacity bulk service queue with single vacation and Markovian arrival process”, International Journal of Stochastic Analysis, Pp.337-357.
11. Jaiswal. N. K., (1960),“Letter to the Editor—Bulk-Service Queuing Problem”, Operations Research, Vol.8(1), Pp.139–143,DOI:10.1287/opre.8.1.139.
12. Kao.C., Li. C.C.and Chen. S.P., (1999),“Parametric programming to the analysis of fuzzy queues”, Fuzzy Sets and Systems, Vol.107(1), Pp.93–100,DOI:10.1016/s0165-0114(97)00295-9.
13. Klir.G. J. and B. Yuan, (1995),“Fuzzy Sets and Fuzzy Logic: Theory and Applications”.
14. Madau. D.P., Feldkamp.L.A., (1996),“Influence value defuzzification method”, Fuzzy Systems, Proceedings of the Fifth IEEE International Conference, Pp.1819 – 1824.
15. Mogharreban.N., DiLalla.L.F.,(2006),“Comparison of Defuzzification Techniques for Analysis of Non-interval Data”, Conference: Fuzzy Information Processing Society,NAFIPS 2006, IEEE Xplore,DOI: 10.1109/NAFIPS.2006.365418
16. Nagoor Gani.A., AshokKumar.V., (2009), “A Bulk Arrival Queuing Model with Fuzzy Parameters and Fuzzy varying Batch Sizes” International Journal of Algorithms, Computing and Mathematics, ©Eashwar Publications, Vol 2(3), Pp. 1-8.
17. Neuts.M.F., (1967), “A General Class of Bulk Queues with Poisson Input”, The Annals of Mathematical Statistics, Vol.38(3), Pp.759–770, DOI:10.1214/aoms/1177698869.
18. Palpandi.B,Geetharamani.G., (2013),“Evaluation of Performance Measures of Bulk Arrival Queue With Fuzzy Parameters Using Robust Ranking Technique”, International Journal of Computational Engineering Research, Vol. 3(10), Pp.53-57.
19. Pinki Gulia,Rakesh Kumar,Gaurav Dhiman, (2024),“ Various Defuzzification Methods for Triangular Fuzzy Numbers Under a Fuzzy Inventory Model”, SSRN Electronic Journal,
20. DOI: 10.2139/ssrn.4502025
21. Roventa.E., Spircu.T., (2003),“Averaging procedures in defuzzification processes”, Fuzzy Sets and Systems, Vol. 136(3), Pp.375–385, DOI: 10.1016/S0165-0114(02)00218-X.
22. Rouhparvar.H., Panahi.A.,(2015), “A new definition for defuzzification of generalized fuzzy numbers and its application”, Applied Soft Computing, Vol. 30,Pp. 577–584 .
23. Sasikala.S., Indhira.K., (2016), “BULK SERVICE QUEUEING MODELS – A SURVEY”, International Journal of Pure and Applied Mathematics, Vol.106(6), Pp.43-56,DOI: 10.12732/ijpam.v106i6.6.
24. Stephen Dinagar, Kamalanathan, Rameshan,(2017),“Sub Interval Average Method for Ranking of Linear Fuzzy Numbers”, International Journal of Pure and Applied Mathematics, Vol.114(6),
25. Sundareswari.K., (2026), “Exploring Nonlinear Dynamics of Information Flow in Distributed Academic and Library Service and Information Source Networks”, Applied Nonlinearity in Science and Technology, Pp.9-18, https://appliednonlinearity.com/index.php/pub/article/view/16.
26. Takagi.H., Takahashi.Y., (1991), “Priority queues with batch Poisson arrivals”, Operations Research Letters, Vol.10(4), Pp.225–232*,*DOI:10.1016/0167-6377(91)90063-u.
27. Van Leekwijck.W., Kerre.E.E.,(1999), “Defuzzification: criteria and classification,” Fuzzy Sets and Systems, Vol. 108(2), Pp.159–178, DOI: 10.1016/S0165-0114(97)00337-0.
28. Wang.Y.M., Yang.J.B., Xu.D.L. and Chin.K.S.,(2006), “On the centroids of fuzzy numbers,” Fuzzy Sets and Systems, Vol. 157(7), Pp.919–926, DOI: 10.1016/j.fss.2005.11.006.
29. Zadeh.L.A., (1965), “Fuzzy sets. Information and Control”, Vol.8(3), Pp.338–353, DOI:10.1016/s0019-9958(65)90241-x.